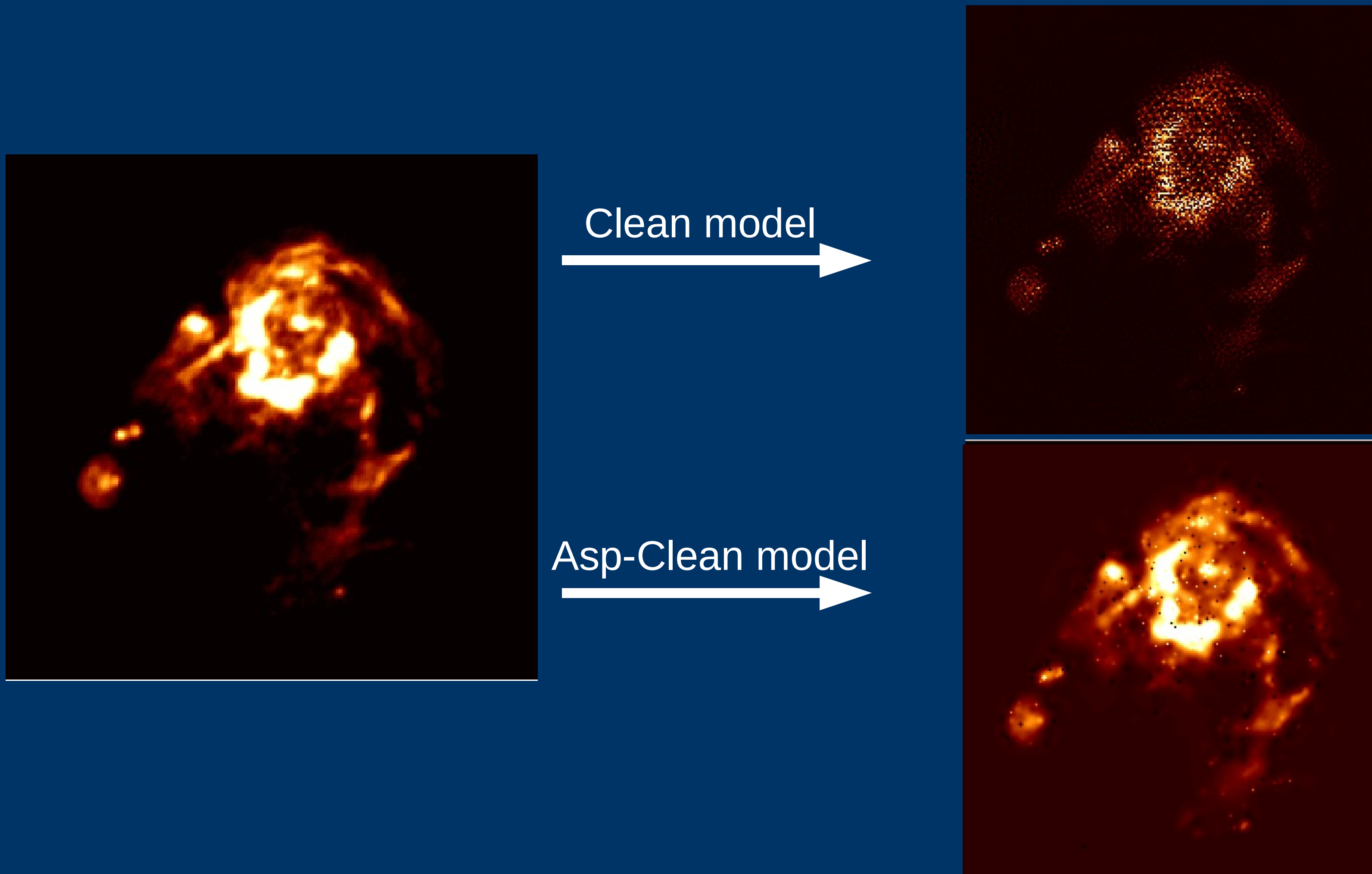
Scale sensitive deconvolution



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Deconvolution

- Deconvolution is a search for I^M which solves the normal equation $A^T V = A^T A\, I^M + A^T A\, N$

$$I^D = PSF * I^M + PSF * I^N$$

- Assuming N a gaussian R.V., χ^2 is the optimal estimator.
- Deconvolution: $Minimize: \chi^2 = [V - AI^M]^T \Lambda [V - AI^M]$
  - Update direction: $\nabla \chi^2 = -2 I_i^R \nabla I^M$
  - Update current model: $I_{i+1}^M = T(I_i^M, \nabla \chi^2)$
  - Compute $I_{i+1}^R = I^D - PSF * I_{i+1}^M$
  - If I^R does not satisfy stopping criteria, repeat

Fundamental issues

- Pixel-to-pixel noise is not independent
  - $I^d = BI^o + BI^N$
  - Neighboring pixels are not independent due to finite resolution

- Extended emission *naturally* couples large number of pixels

- Consequently, the effective number of degrees of freedom (DoF) are significantly less than the number of pixels with significant emission
  - Can lead to “over fitting”, or
  - Low level residuals

- Pixelation

Scale-less deconvolution

- Each pixel is treated as a DoF
- Search space constraints by user supplied “boxes”

$$I^M = \sum_k A_K \delta(x_k)$$

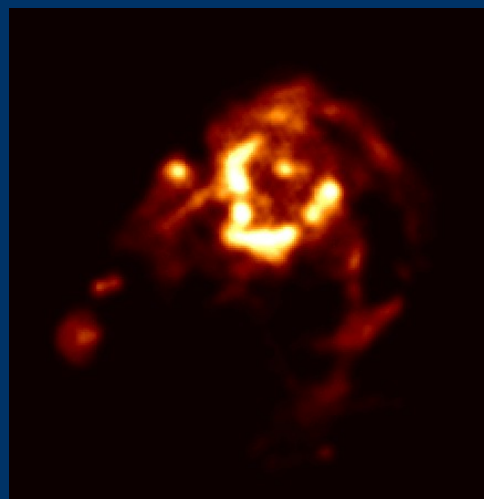
- Clean (CS-Clean): A Steepest Descent algorithm
  - Dimensionality of the search space is equal to number of pixels in the “boxes”
  - Approx. update direction: $\nabla \chi^2 = Residual\ Image(I^R)$
  - Iteratively take a step along the axis with the highest derivative: $I^M_{i+1} = I^M_i + g[max : I^R_i]$
- MEM: Constrained minimization:
  - Minimize: $H - \lambda \chi^2$ *where* λ *is a Lagrange multiplier*
  - ***H*** is the Entropy function

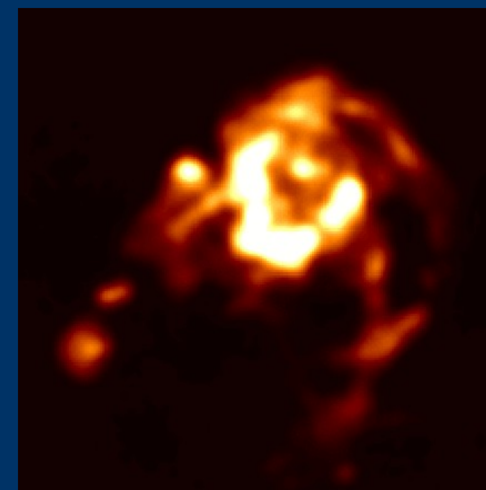
Reconstructed images and residuals

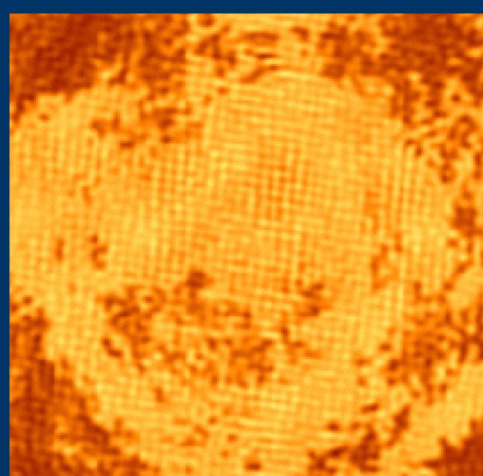
CS-Clean

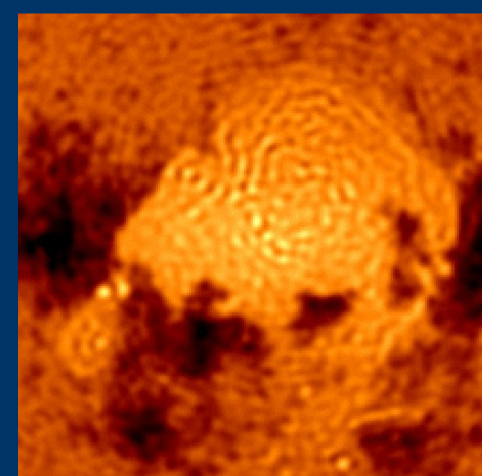
Niter = 60K

MEM









Separation of signal and noise

$$I^d = BI^o + I^N$$

- The Dirty Image = True image corrupted in a deterministic way by the PSF + non-deterministic term
- Peak value is used to separate the two terms
  - This works well in the regime where the first term dominates

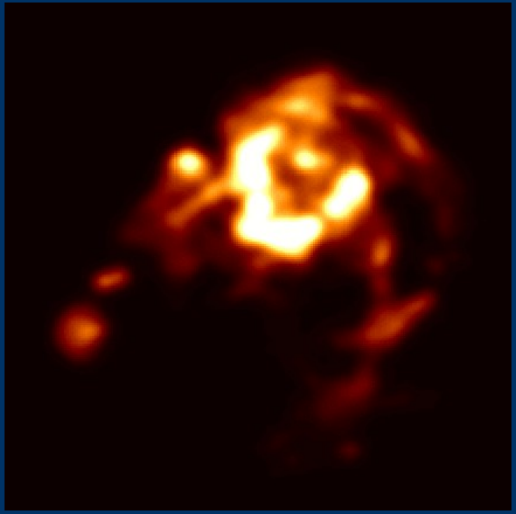
- With low level large scale emission, the peak of the two terms is comparable
- Need to incorporate explicit prior knowledge to separates signal from noise.

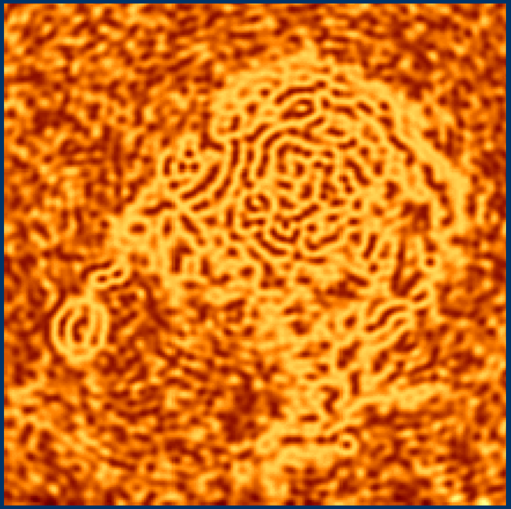
- Scale *fundamentally* separates signal from noise:
  - All scales in I^N < ~PSF Main-lobe
  - All scales in BI^{True} > ~PSF Main-lobe

Scale sensitive deconvolution

- Incorporate the fact that scale of emission changes across the image *explicitly* in the algorithms
  - Discover the local scale
  - Compute its effect in the rest of the image
- MS-Clean
  - Decompose the image using a fixed set of scales
  - Retains the shift-and-subtract nature of Clean algorithm
  - Matched filtering technique
  - However, assumes an orthogonal search space
    - No interaction between scales

Model image:
~15K comps.

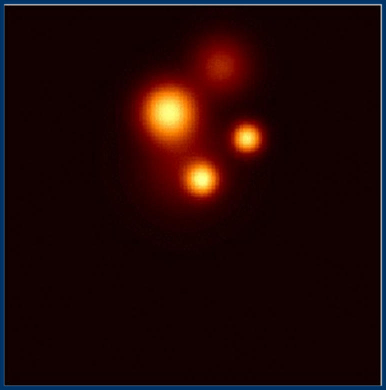




Residual
Image

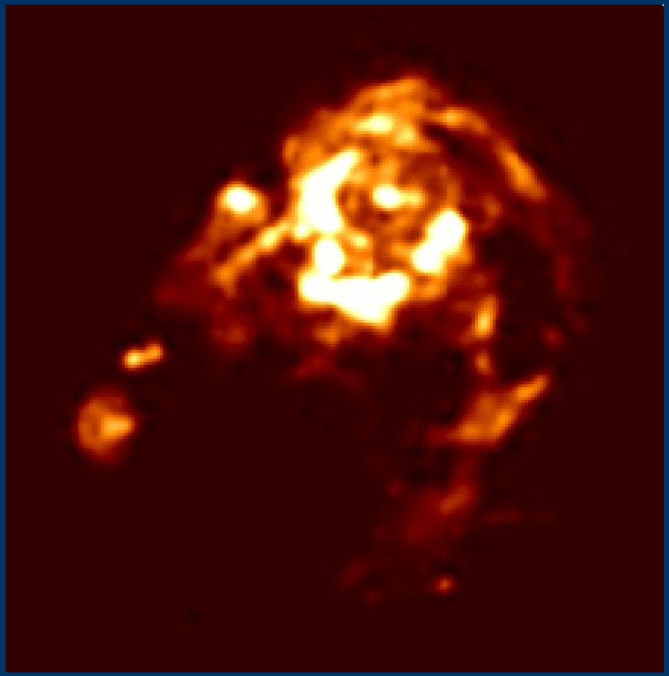
Scale sensitive deconvolution

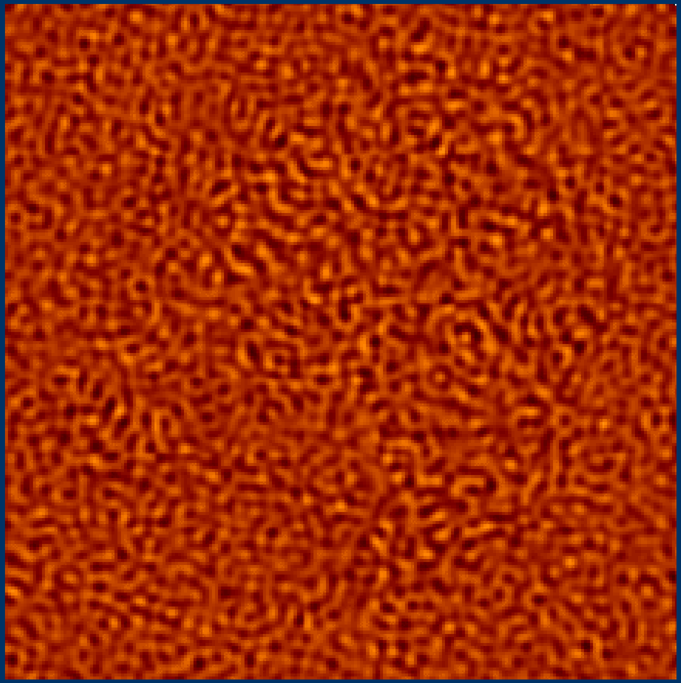
- Search space is *not* orthogonal in general
- Fixing the set of allowed scales is non-optimal.



- Adaptive Scale Pixel (Asp)-Clean
  - Incorporate interaction between scales
  - Discover the local scale adaptively.
  - Use local scale as a the “size” of the logical “pixel”

Model image:
~1K comps.





Residual
Image

Challenge

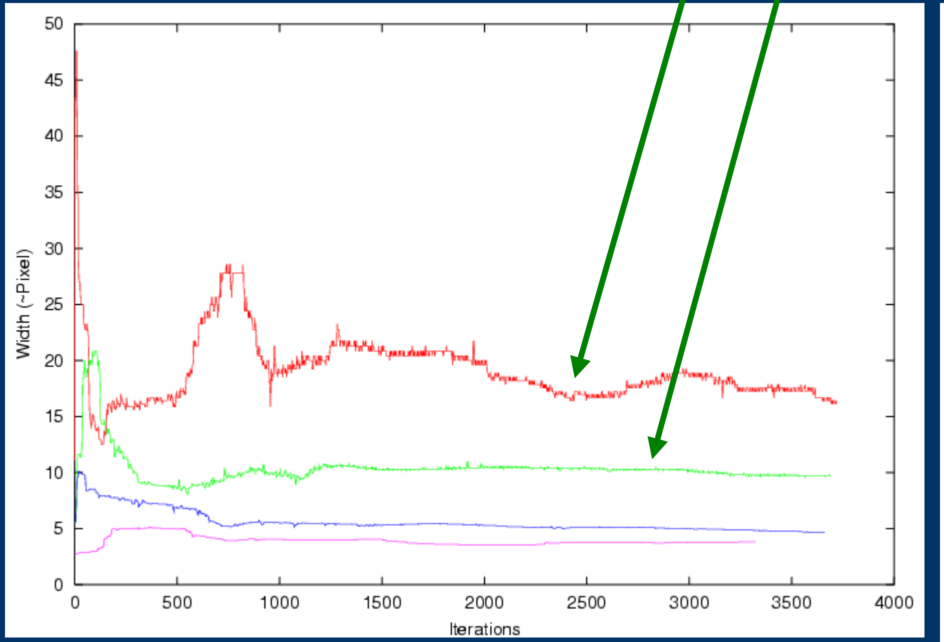
- Incorporation of interaction between scales
  - Computing scales strongly with image complexity
- Computing the effects for the entire image in each iteration is expensive

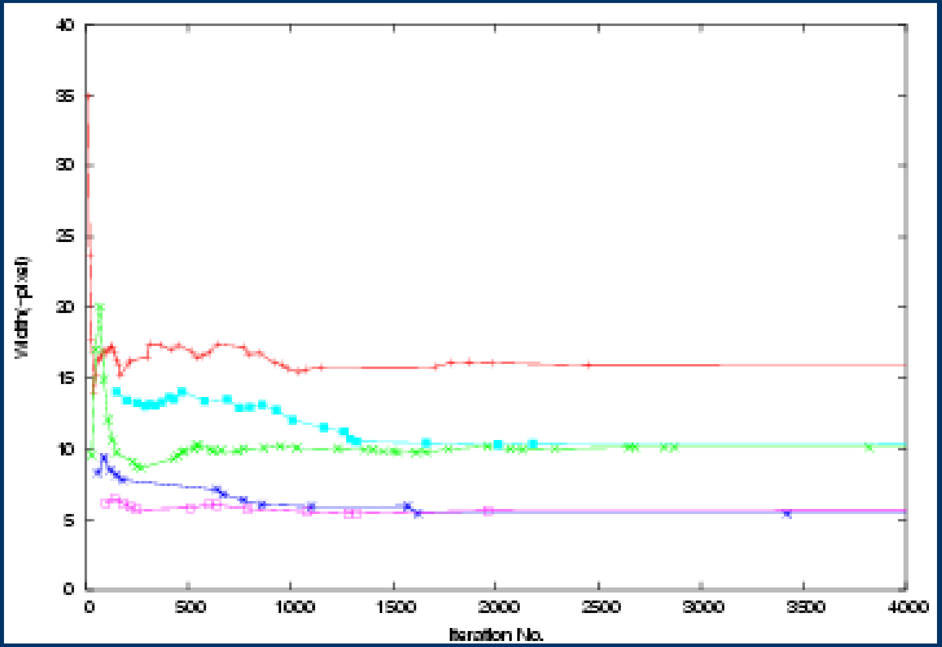
- Acceleration methods
  - At each iteration, make a set of active Asps which have maximal impact in the current iteration
  - Restrict the search at each iteration to this sub-space
    - Keep the dimensionality of the search space in control

Asp-Clean acceleration

Fig 1: All Aspen are kept in the problem for all iterations. Scales all Aspen evolve as a function of iterations.
But note: Not all Aspen evolve significantly at all iterations.

Fig 2: The active-set is determined by thresholding the first derivative. Only those Aspen, shown by symbols, are kept in the problem which are likely to evolve significantly at each iteration.





Clean, MEM, MS-Clean, Asp-Clean

- I^d-BI^M