Fourier phase analysis in radio-interferometry

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Imaging and Calibration Algorithms for EVLA, e-MERLIN, ALMA and VLBI
Oxford e-Research Centre
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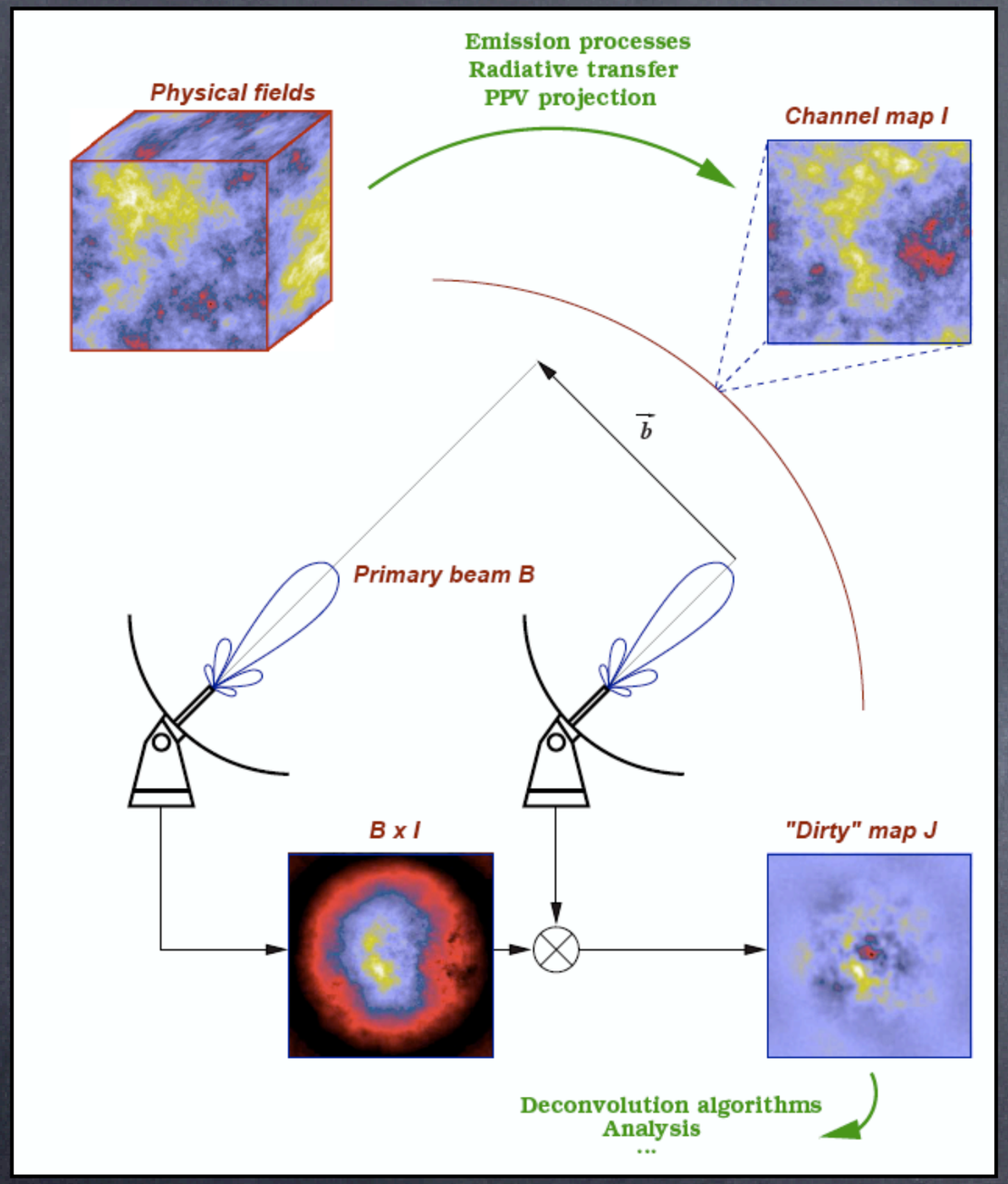


- *Projection on a PPV hybrid space*
- *Antenna pairs measure correlations*
- *Primary beam attenuation*
- *Incomplete sampling*

$$J = T_F^{-1}[C \times T_F[B \times I]] = T_F^{-1}[V]$$

How is structure encoded in interferometric images ?

Statistical measures

- Second order structure function $S_F(\boldsymbol{r}) = \left\langle [F(\boldsymbol{x}+\boldsymbol{r}) - F(\boldsymbol{x})]^2 \right\rangle_{\boldsymbol{x}}$
- Autocorrelation function $A_F(\boldsymbol{r}) = \langle F(\boldsymbol{x}+\boldsymbol{r}) F(\boldsymbol{x}) \rangle_{\boldsymbol{x}}$
- Power spectrum $P_F(\boldsymbol{k}) = \left| \tilde{F}(\boldsymbol{k}) \right|^2$

Direct numerical approach

Model field S_0, A_0, P_0 ⟶ Instrument simulator ⟶ "Observed" field S, A, P

What statistical tools are the most reliable ?

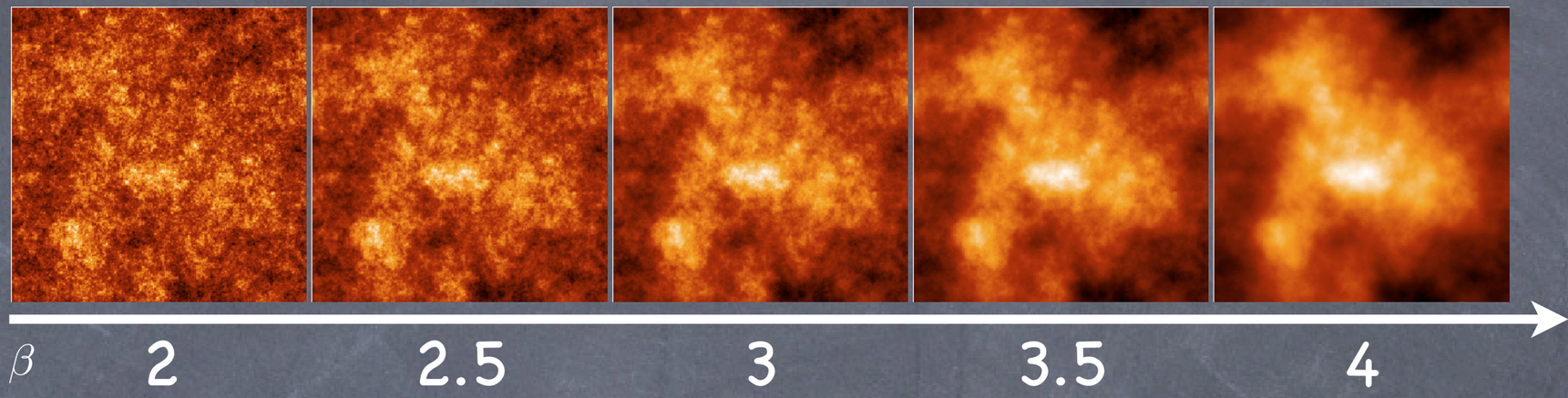
fractional Brownian motion fields

Statistical behaviour

$$S(\boldsymbol{r}) \propto |\boldsymbol{r}|^{2H} \qquad H \in [0,1]$$

$$P(\boldsymbol{k}) \propto |\boldsymbol{k}|^{-\beta} \qquad \beta = 2H + n$$

Fully random Fourier phases



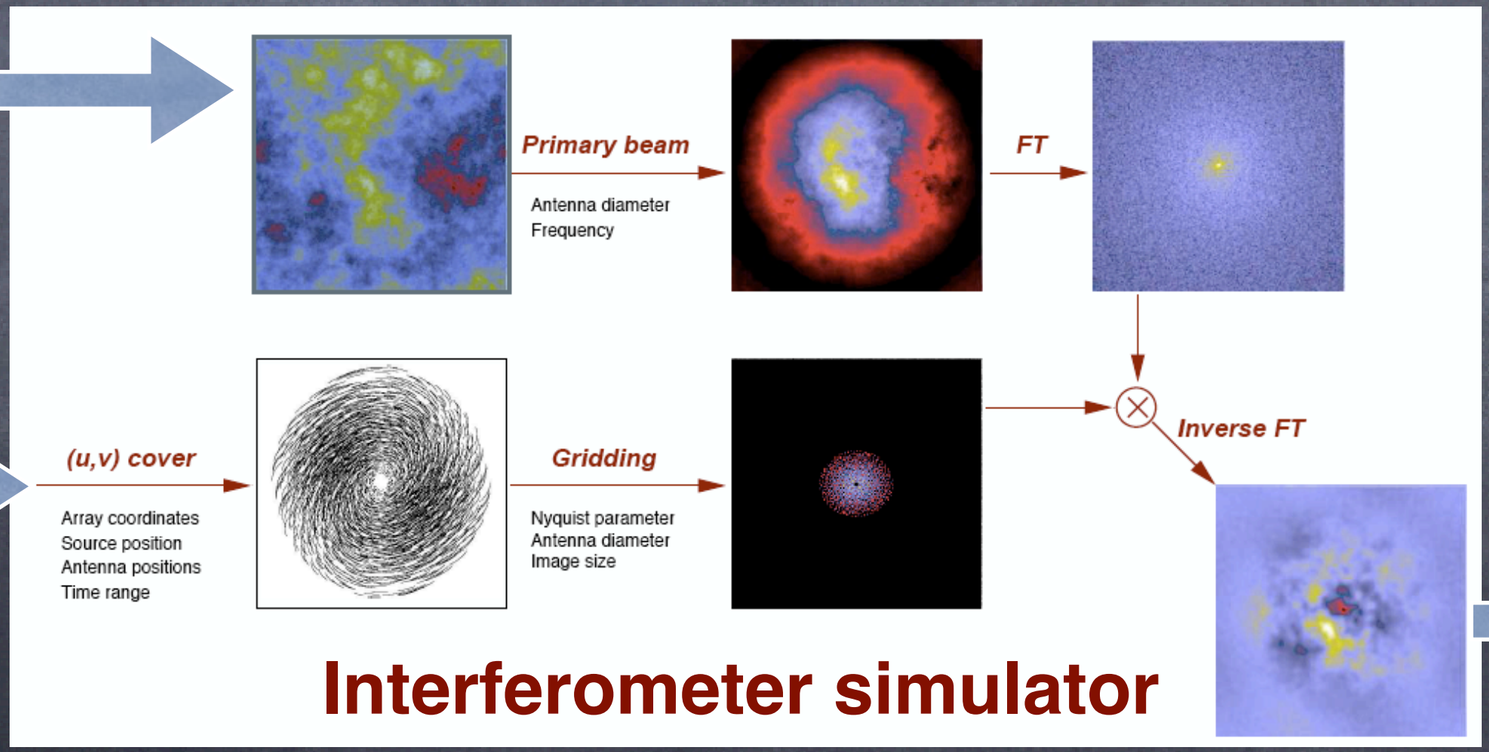
Numerical implementation

- Ease of generation in Fourier space
- Models of the diffuse interstellar medium

(Stutzki et al., 1998; Bensch et al., 2001; Brunt & Heyer, 2002; Miville-Deschênes et al., 2003; Levrier, 2004)

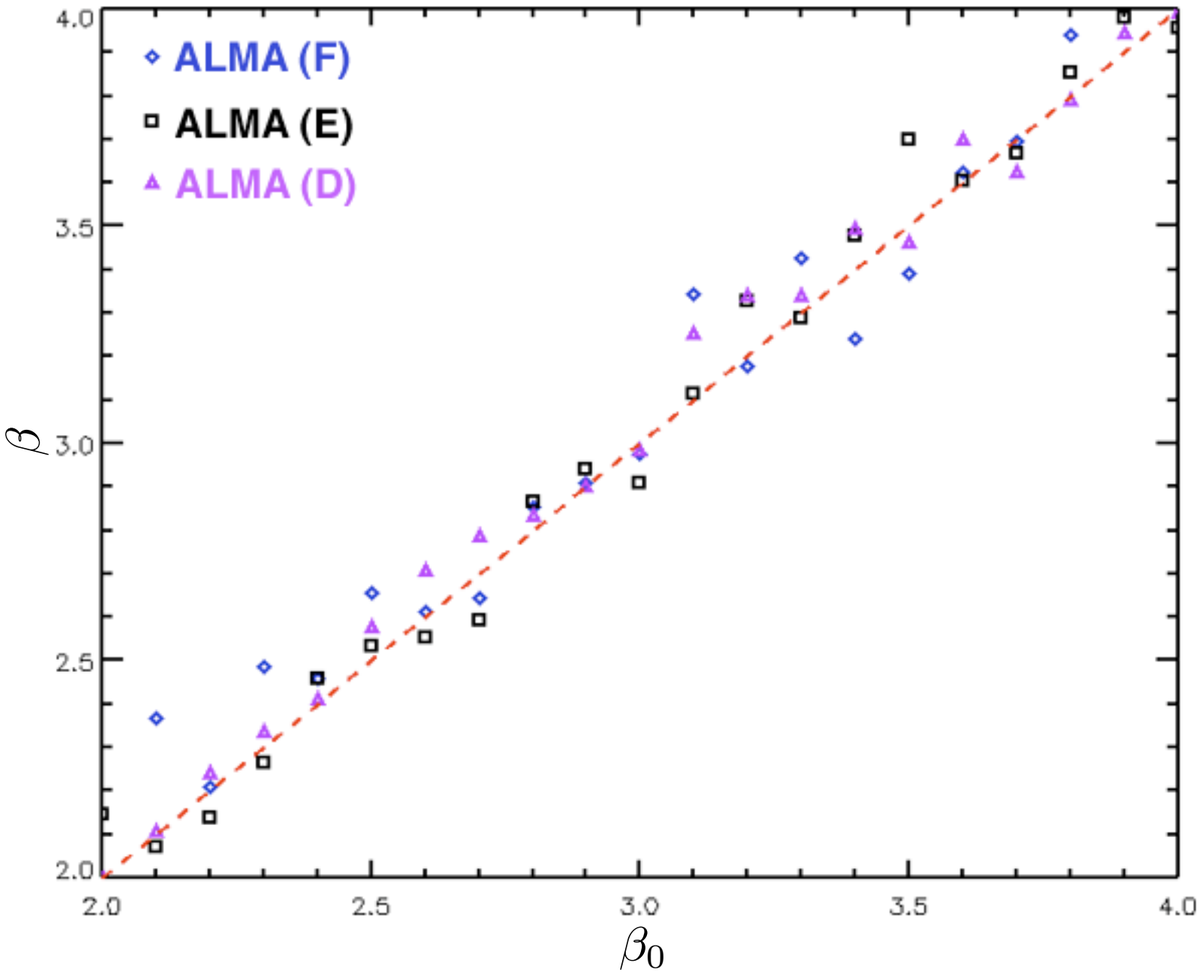
fBm model fields

ALMA
VLA
PdBI
...

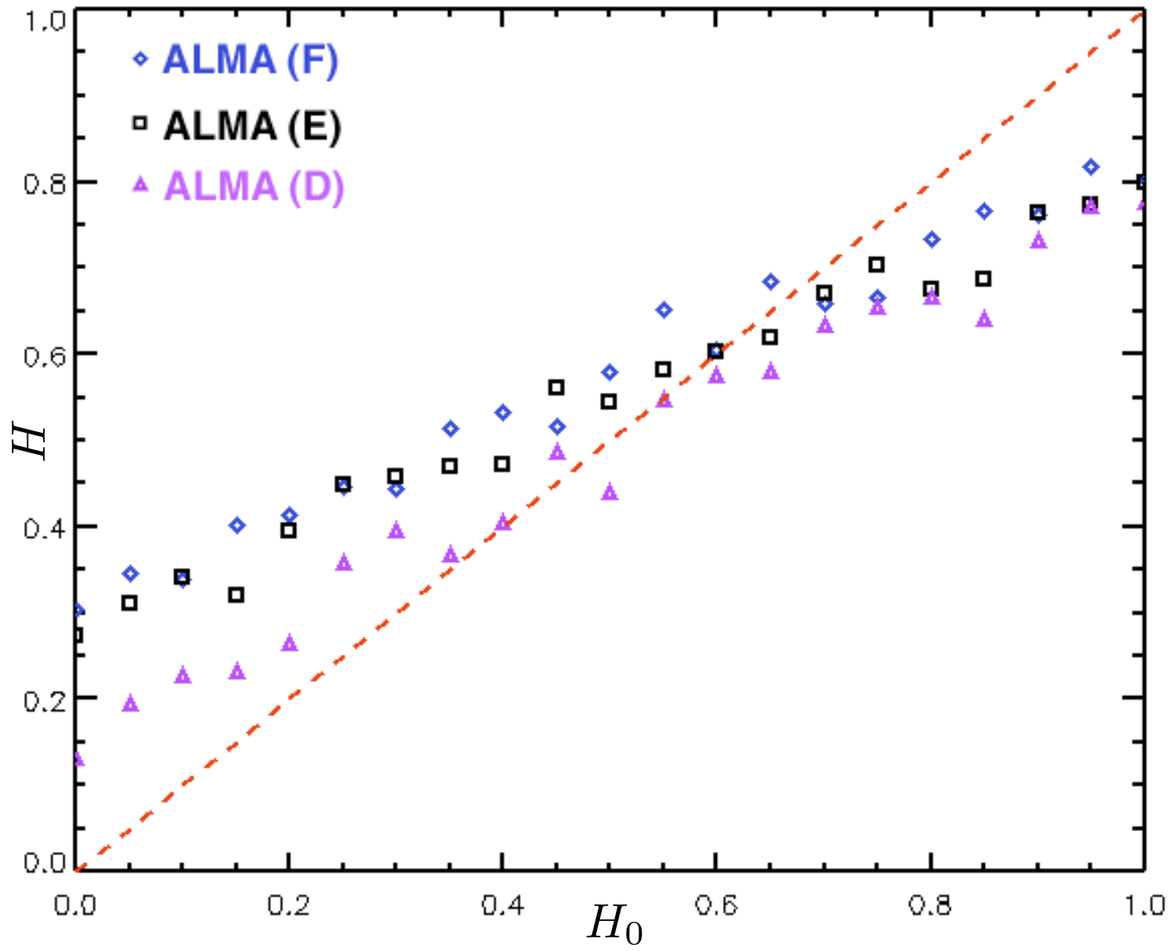


Statistical measures

Power spectrum



Structure function

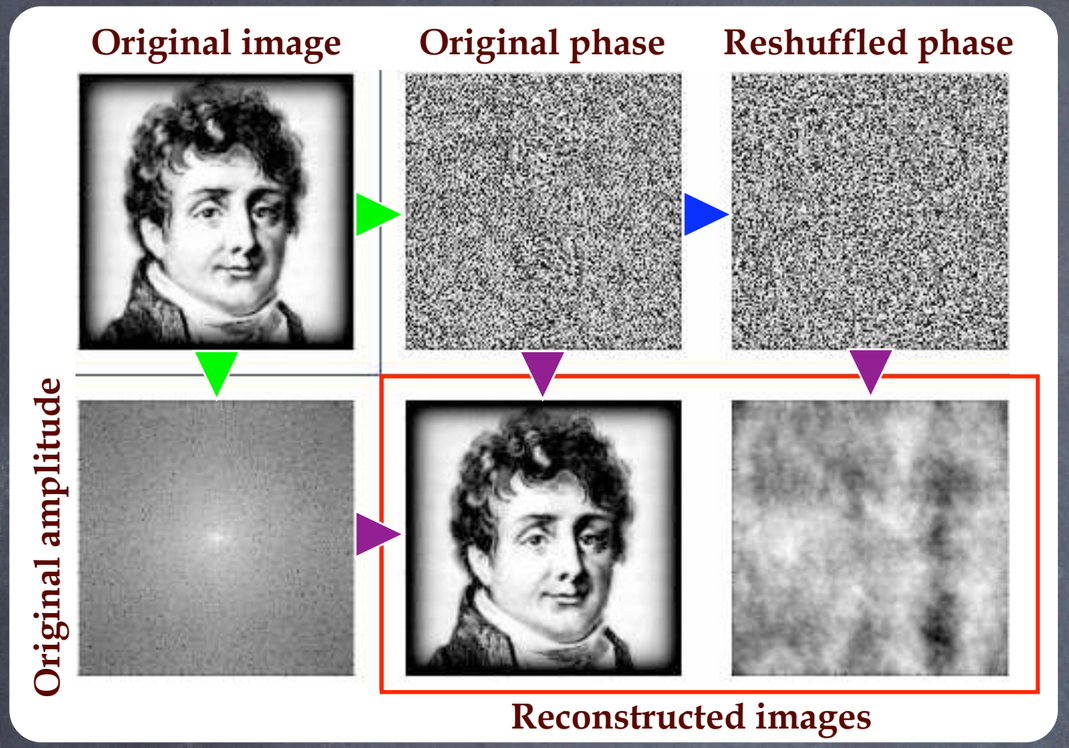


Adaptation of statistical tools to the measurement space

Structure function ⟷ Single dish (Bensch et al., 2001)

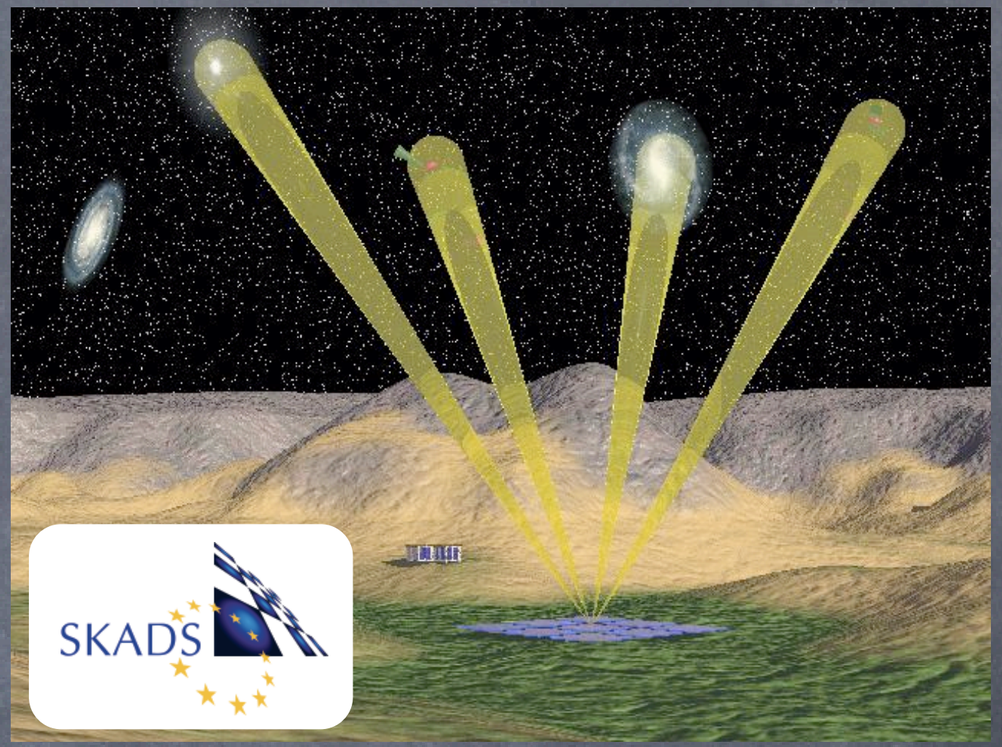
Power spectrum ⟷ Aperture synthesis (Levrier, PhD Thesis, 2004)

But power spectra only make use of Fourier amplitudes, not phases...



Reshuffling of the Fourier phases

Loss of structural information



Phased planar arrays

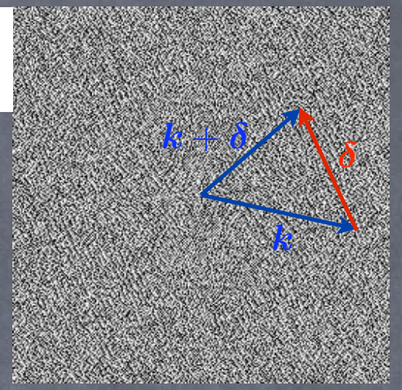
Multibeaming

Statistics of phase increments

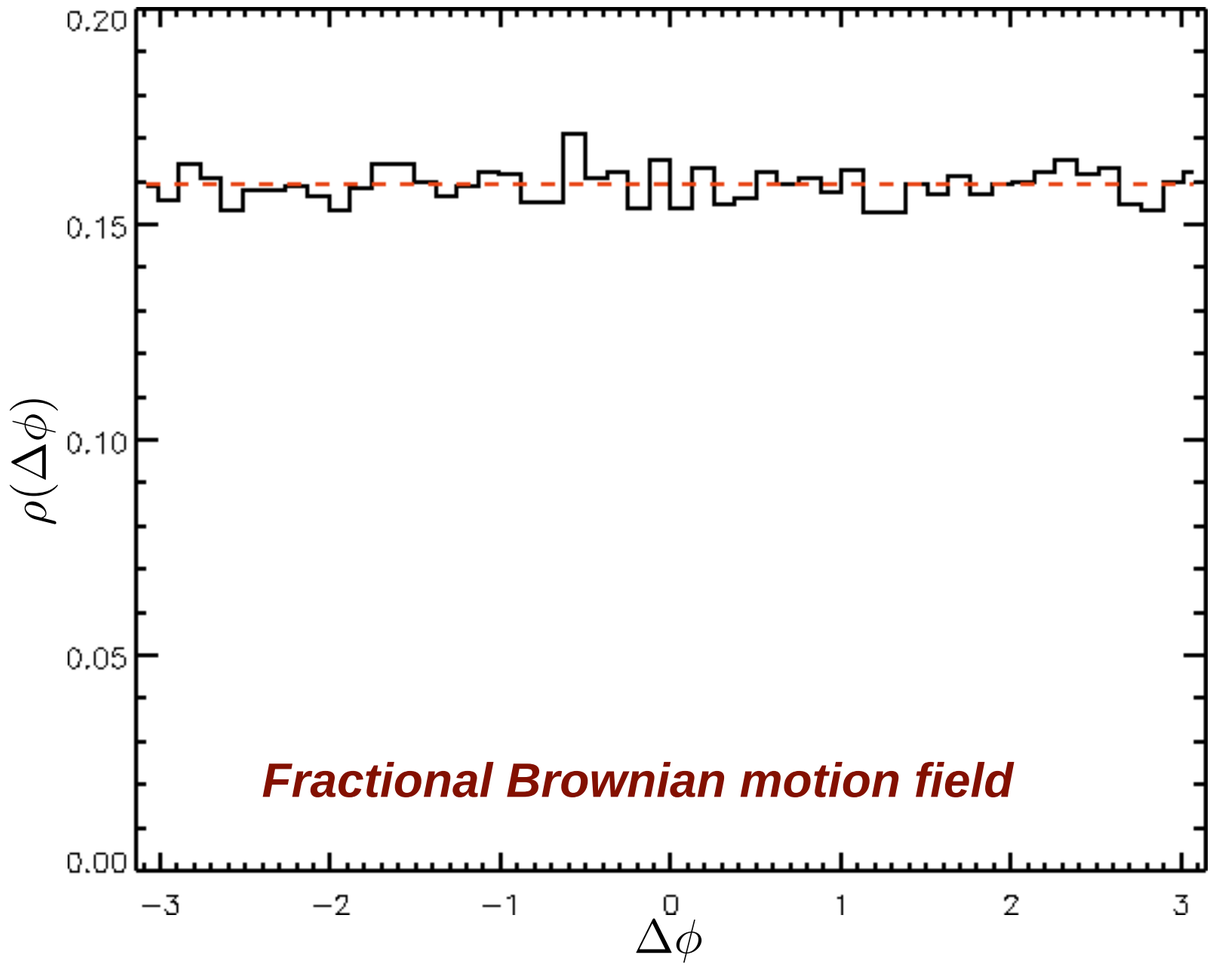
$$\Delta\phi(\boldsymbol{k};\boldsymbol{\delta}) = \phi(\boldsymbol{k}+\boldsymbol{\delta}) - \phi(\boldsymbol{k})$$

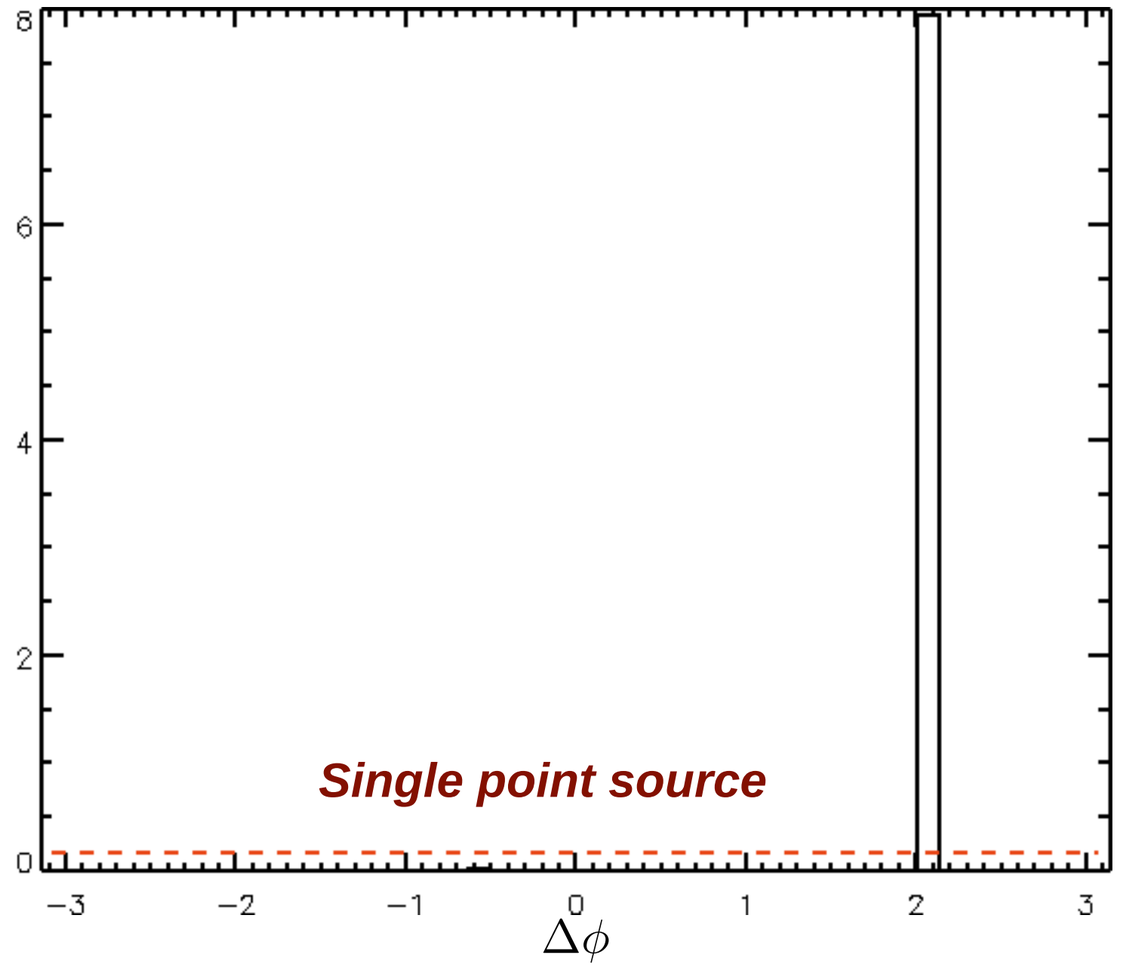
- Statistics of phase increments should trace structure lost in the reshuffling
- Probability distribution functions $\rho(\Delta\phi)$ over the wave vector are functions of the lag vector

Wave vectors
Lag vector

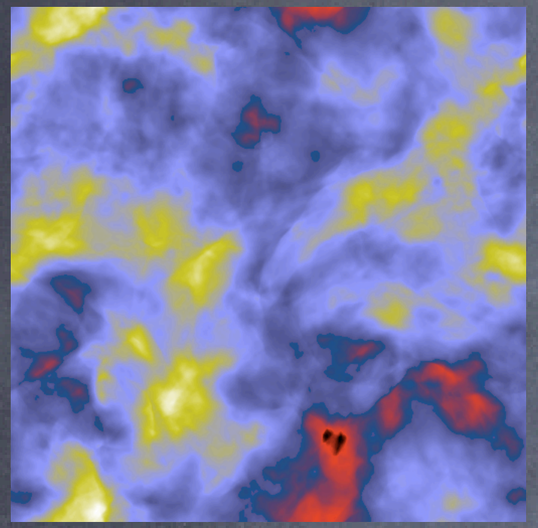


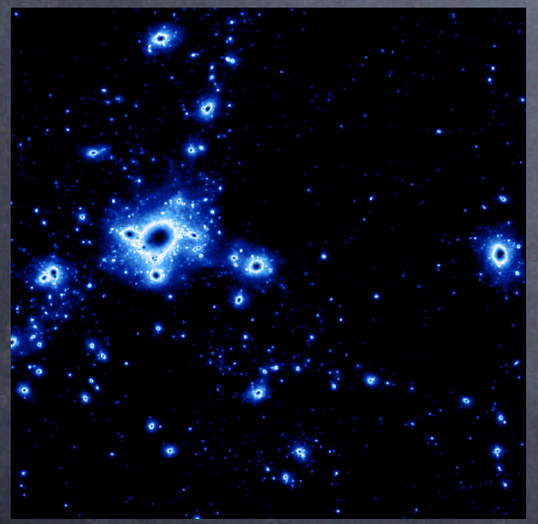
Limiting cases



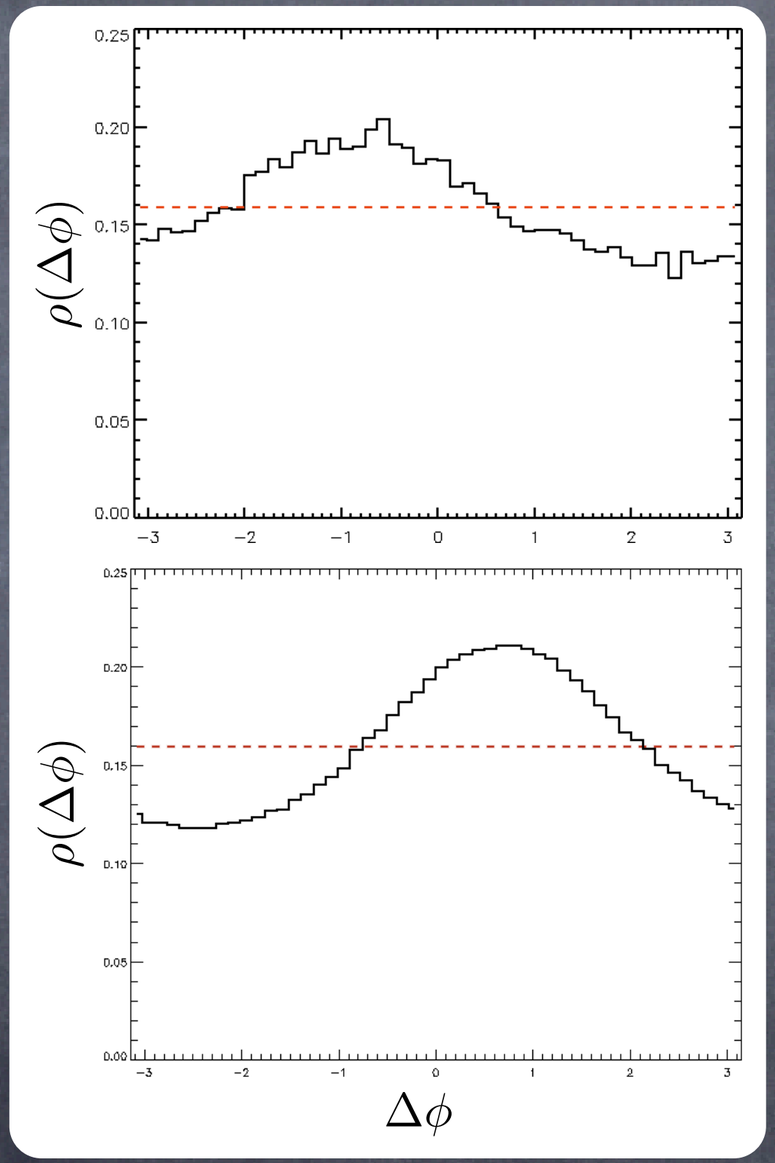


Compressible hydrodynamical turbulence simulation (Porter et al., 1994)



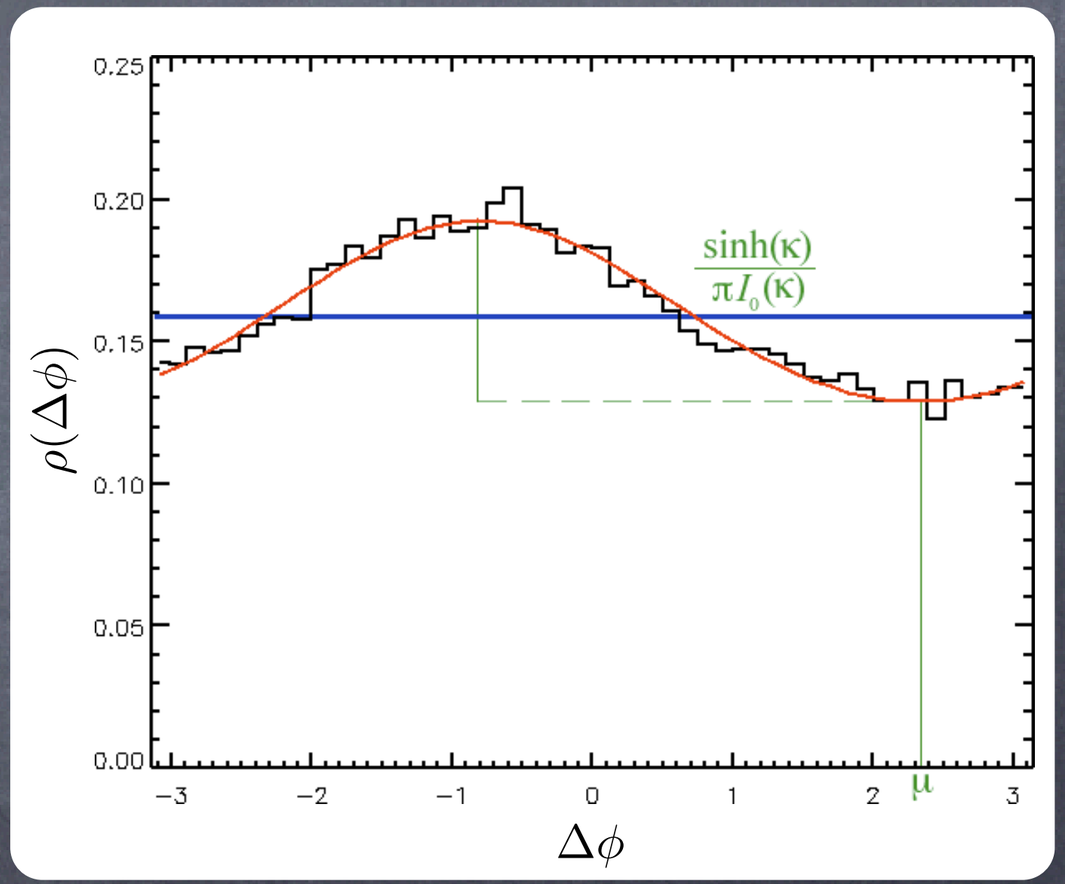


Gravitational clustering simulation (Horizon project)



General form:
Von Mises distribution
(Watts et al., 2003)

$$\rho(\Delta\phi) = \frac{1}{2\pi I_0(\kappa)} e^{-\kappa(\cos\Delta\phi - \mu)}$$



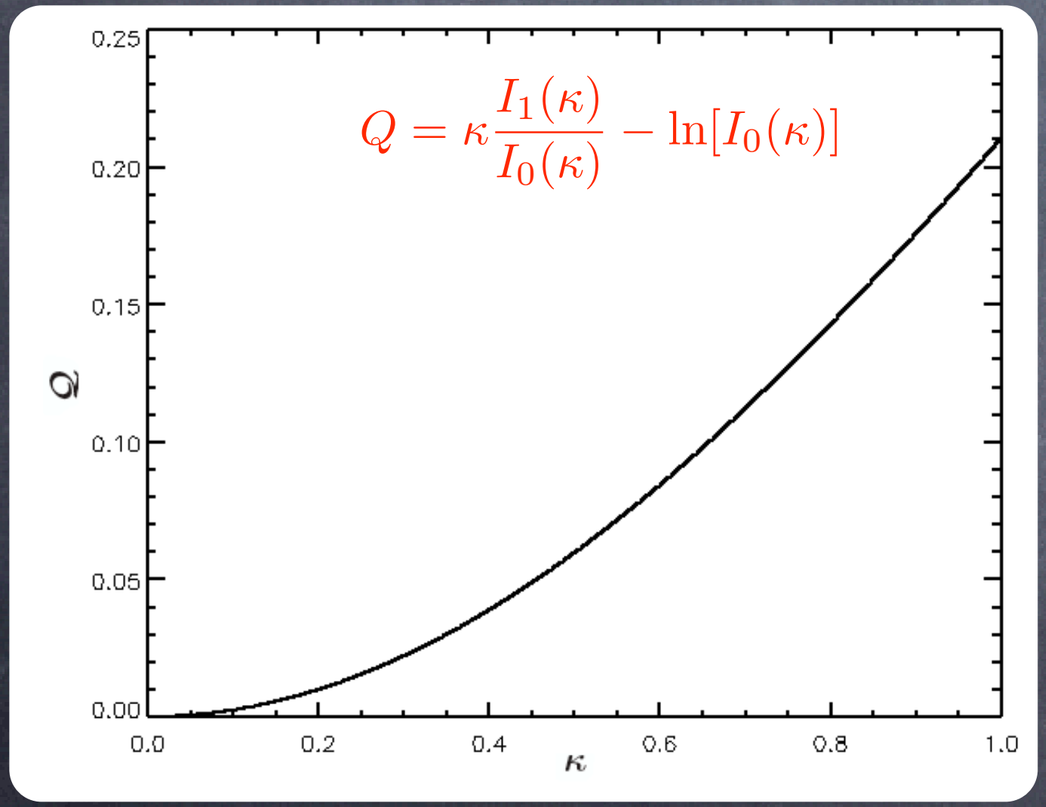
Phase entropy and phase structure quantity

(Polygiannakis & Moussas, 1995)

$$\mathcal{S}(\boldsymbol{\delta}) = -\int_{-\pi}^{\pi} \rho(\Delta\phi)\ln[\rho(\Delta\phi)]\mathrm{d}\Delta\phi$$

$$\mathcal{Q}(\boldsymbol{\delta}) = \ln(2\pi) - \mathcal{S}(\boldsymbol{\delta}) \geqslant 0$$

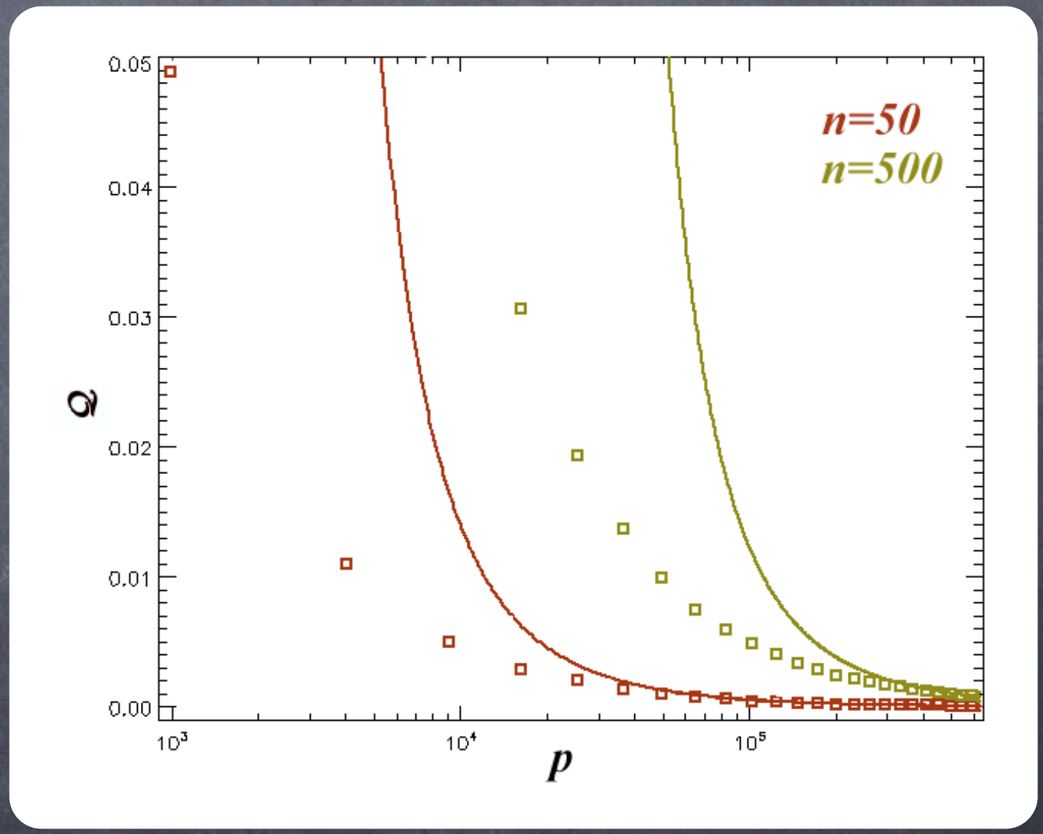
- Fractional Brownian motion : $\mathcal{Q}(\boldsymbol{\delta}) = 0$
- Point source : $\mathcal{Q}(\boldsymbol{\delta}) = \infty$
- Turbulence simulation : $\mathcal{Q}(\boldsymbol{\delta}) \sim 10^{-2}$
- Gravitational clustering simulation : $\mathcal{Q}(\boldsymbol{\delta}) \sim 10^{-1}$



Statistical noise on finite-sized images

- May lead to false detection of phase structure
- Requires an estimate of x so that no phase structure implies $\mathcal{Q} < x$
- Depends on number of phase increments p and number of bins n
- Theoretical upper limit x computed from chi-square statistics

(Levrier, Falgarone & Viallefond, 2006)



Configurations

ALMA, VLA, PdBI

Primary beam attenuation

Convolution in Fourier space

Mosaic observations effectively reduce kernel size

Not considered ⟷ Pointlike antennae

Pillbox gridding

Measured phases associated with "wrong" wavenumber

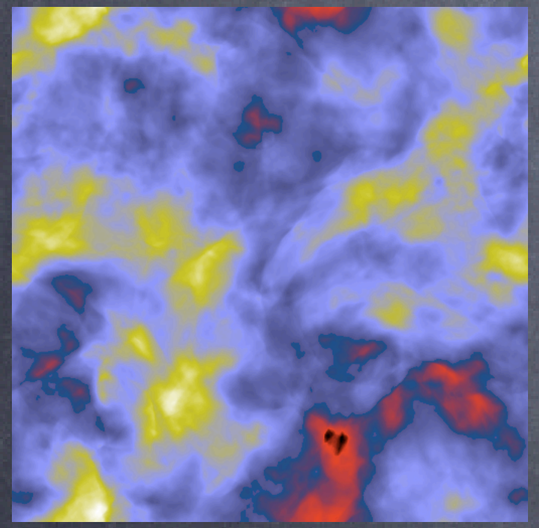
Model brightness distributions already gridded

Not considered ⟷ Phase constant over each pixel

Atmospheric phase noise

Atmospheric turbulence makes phase space- and time-dependent

Considered ... See later



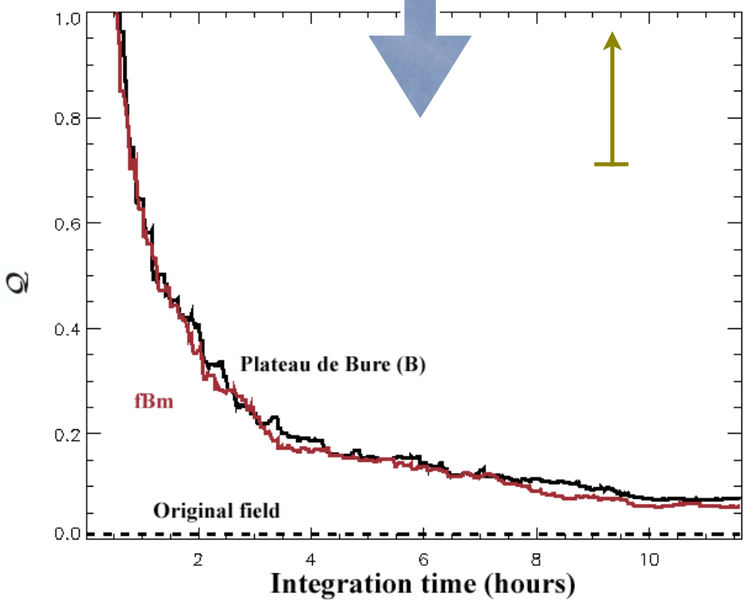
Interferometer simulator

How long does it take to achieve a significant detection of phase structure in this field?
How long does it take to recover the actual phase structure quantity ?
What level of atmospheric turbulence still allows detection of phase structure ?

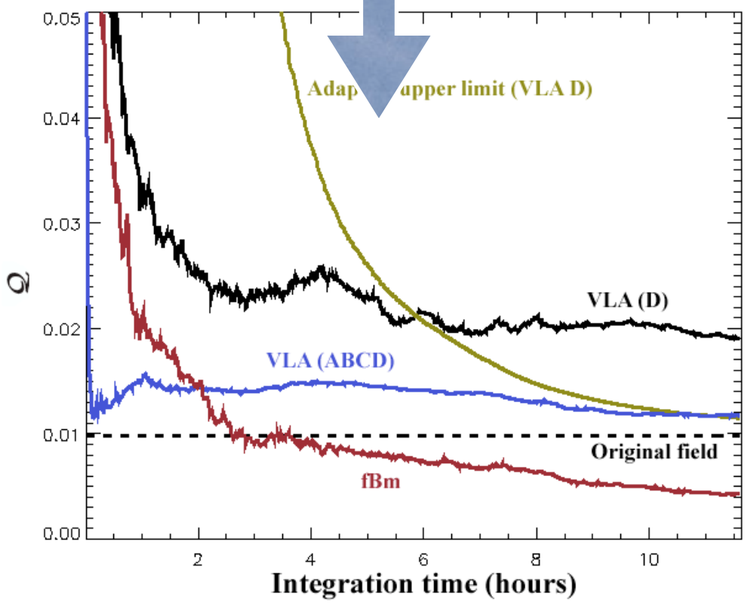
Evolution of measured phase structure quantity as a function of integration time

- Phase structure quantity of the output dirty image with a single configuration
- Phase structure quantity of the output dirty image with a single configuration when input image is a fractional Brownian motion
- Theoretical upper limit for "phase structure quantity noise" with a single configuration
- Phase structure quantity of the output dirty image compiled from multiple configurations
- Phase structure quantity of the input image

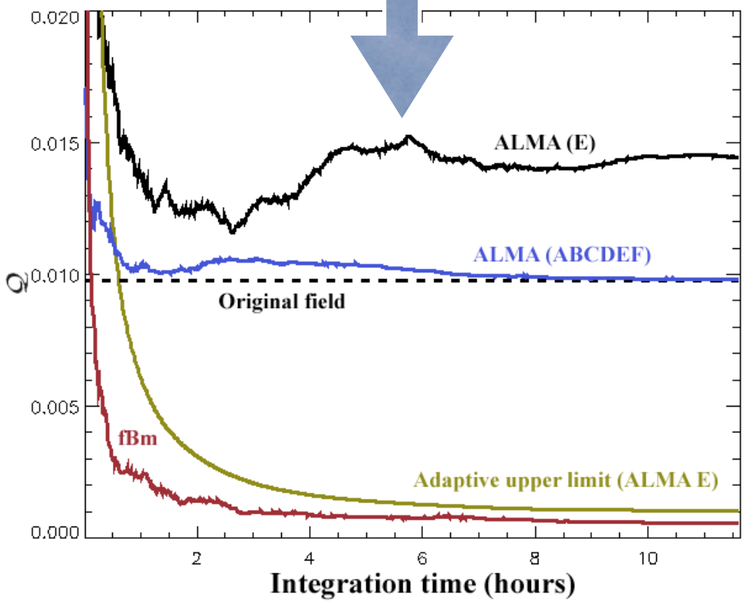
PdBI



VLA



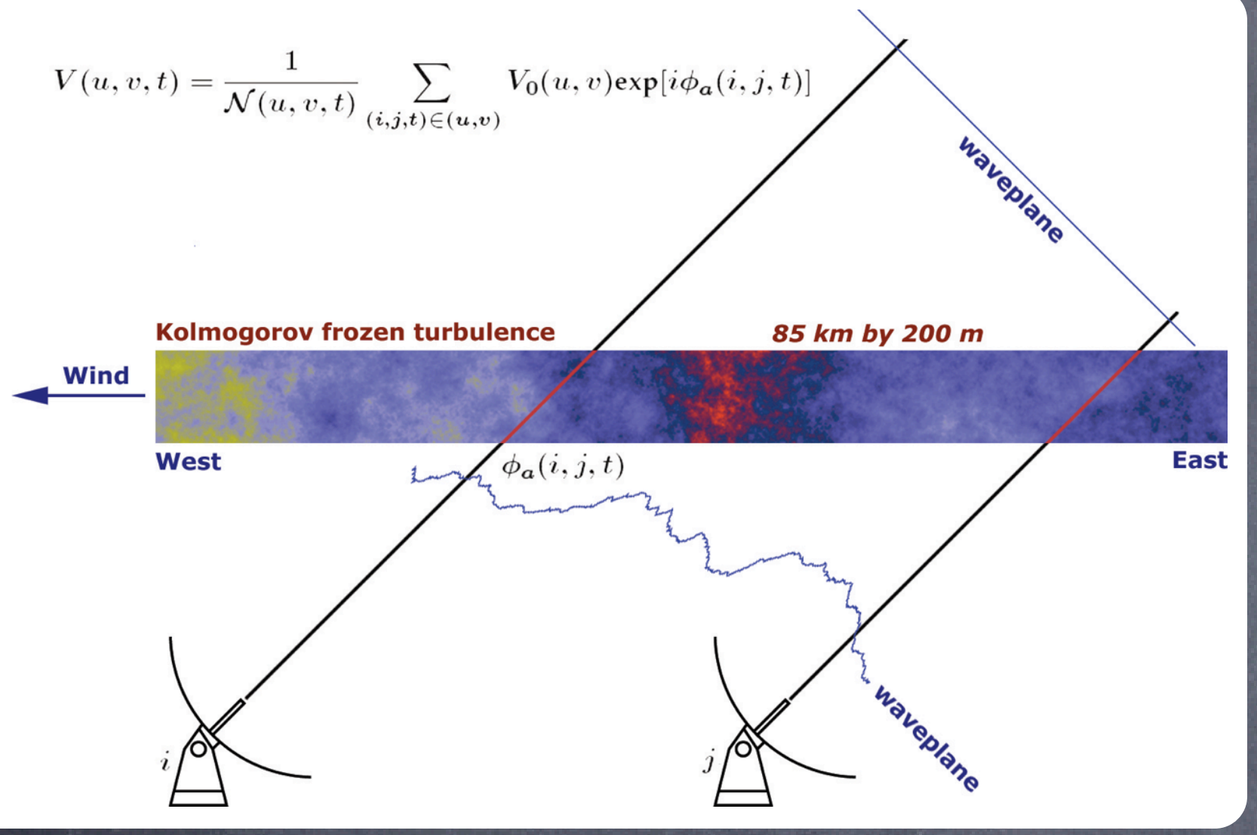
ALMA

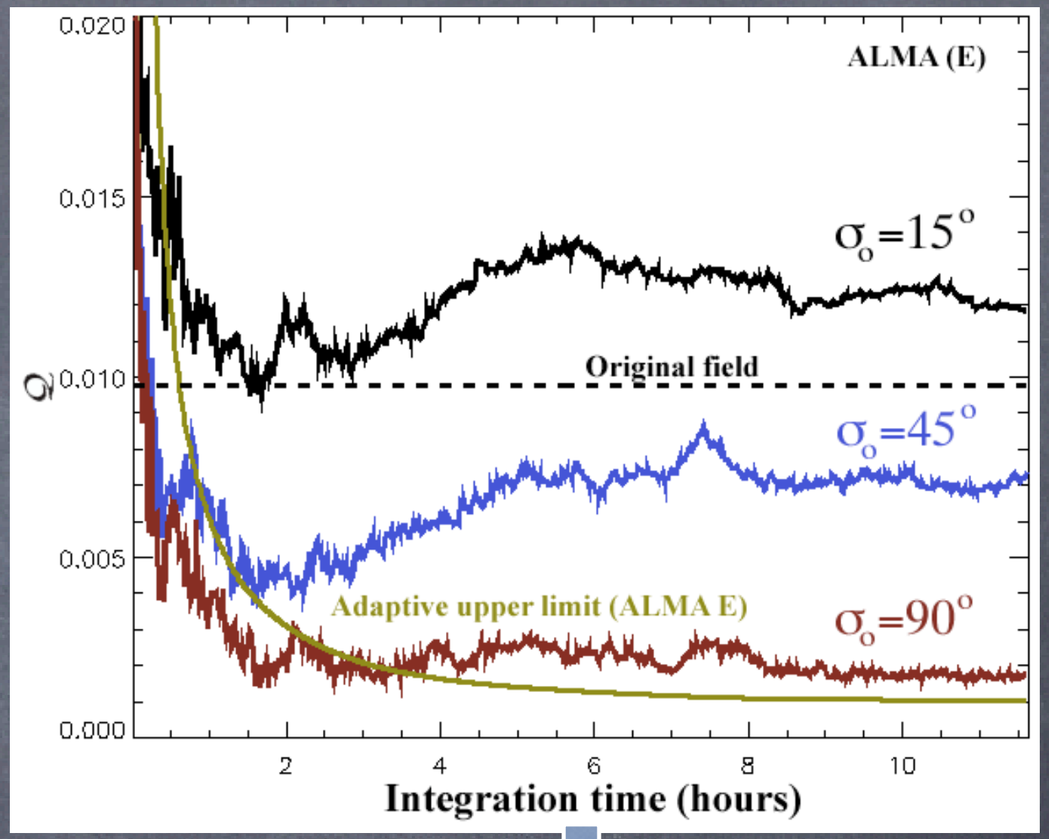


No detection ***Detection*** ***Measurement***

Influence of atmospheric phase noise

$$V(u,v,t) = \frac{1}{\mathcal{N}(u,v,t)} \sum_{(i,j,t)\in(u,v)} V_0(u,v)\exp[i\phi_a(i,j,t)]$$





σ_0 : rms of atmospheric phase delay ϕ_a for a 100-m baseline zenithal observation at 1.3 mm

At Llano de Chajnantor : $\sigma_0 \sim 15^\circ - 60^\circ$

- ***Detection possible without correction***
- ***Measurement requires WVR / FS***

Summary

A new tool to quantify complex structures

Typical values
- ~ 0.01 for compressible hydrdynamical turbulence simulations
- ~ 0.1 for gravitational clustering simulations
- Intermediate values on ISM fields (Taurus Molecular Cloud)

Discrimination of physical processes at work ?

Detection of phase structure
- Requires large number of antennas
- Atmospheric phase noise not critical

Measurement of phase structure
- Requires multiple configurations for optimal Fourier-space coverage
- Correction by water vapor radiometry / fast-switching

May be used in the context of interferometry

Open questions

- Interpretation of phase structure quantities with respect to physical processes
- Weighting by Fourier amplitudes : combination with power spectra
- Allowing for variations of the lag vector : decline of phase structure quantities