# Difference imaging and multi-channel Clean

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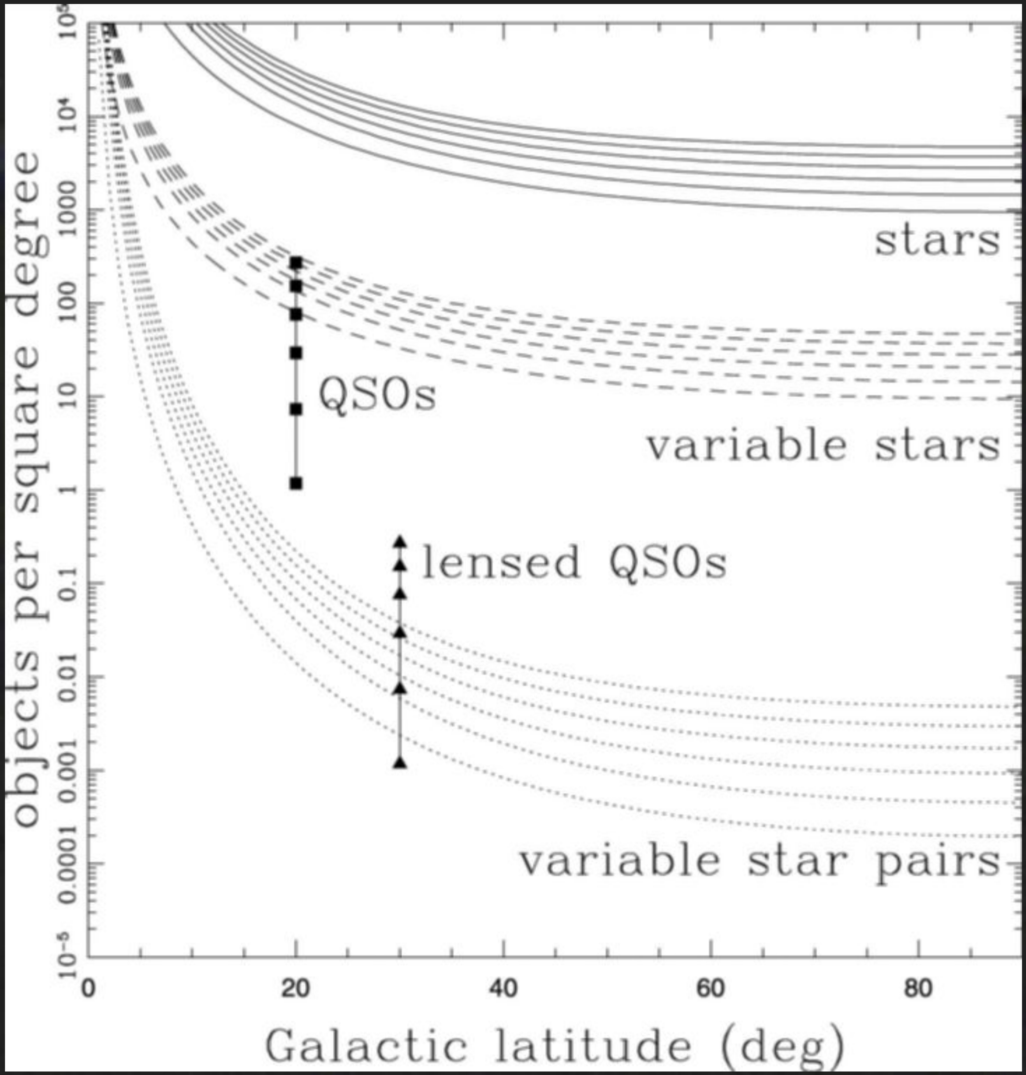
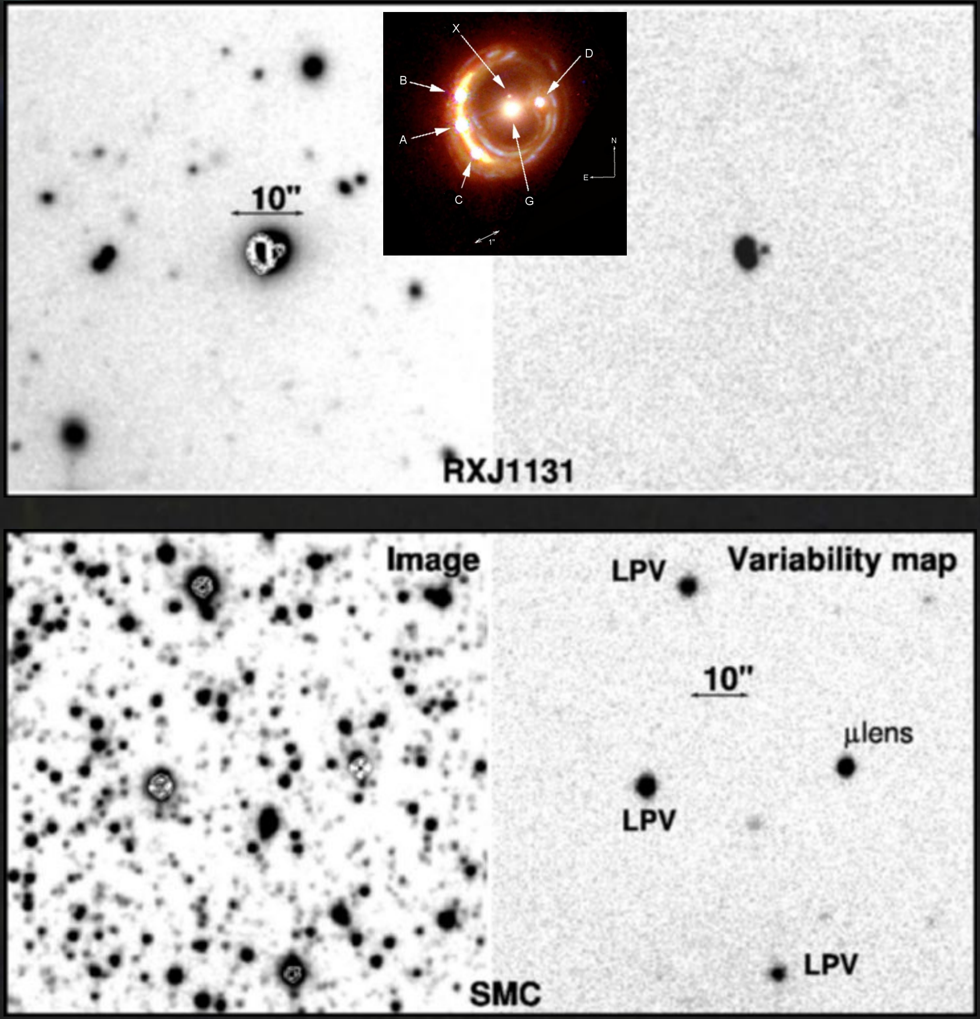
# Finding gravitational lenses through variability

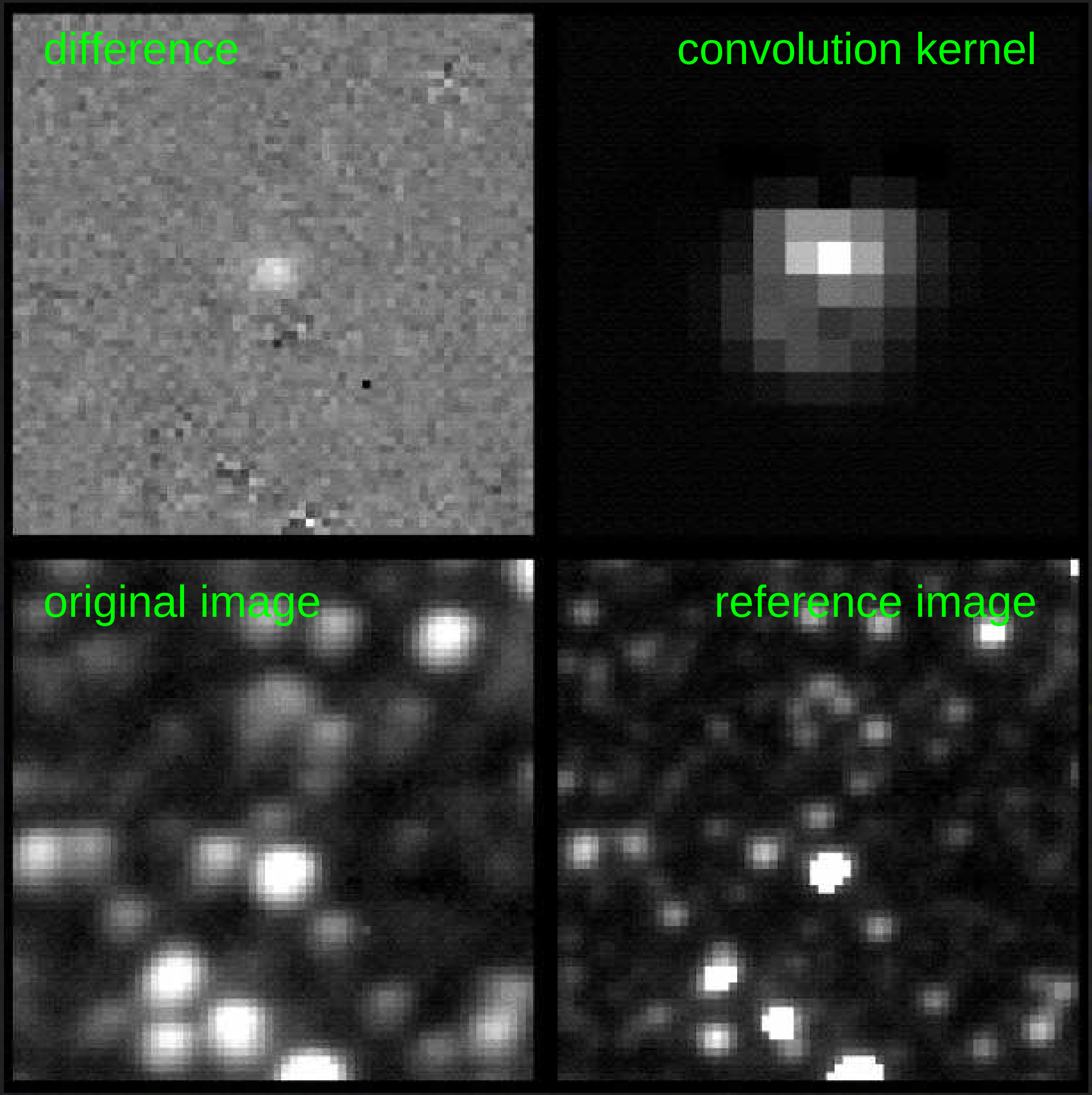



[ *Kochanek et al. (2006)* ]



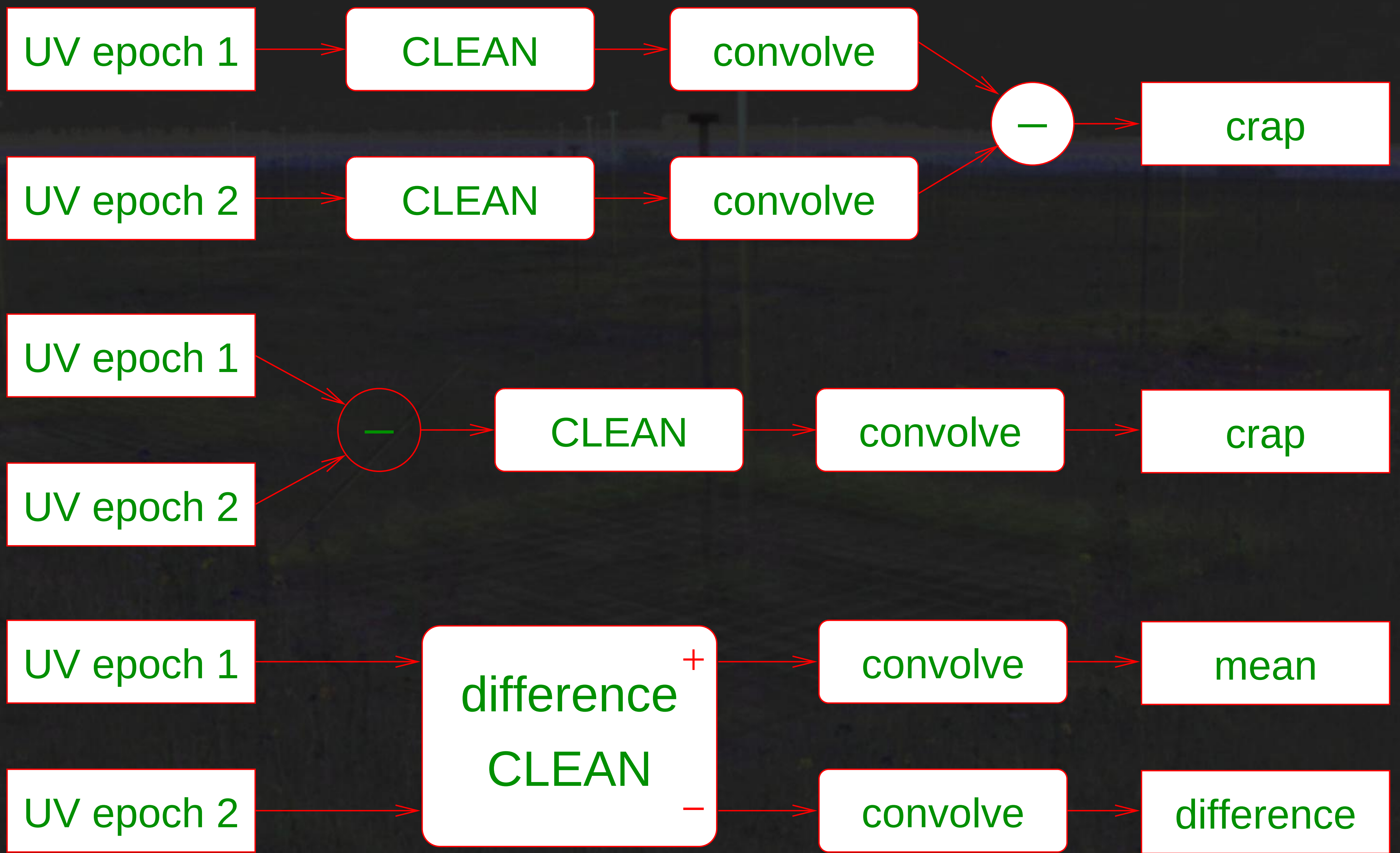

- small field in SDSS: one candidate, no lens [ *Lacki et al. (2008)* ]

# Optical difference imaging

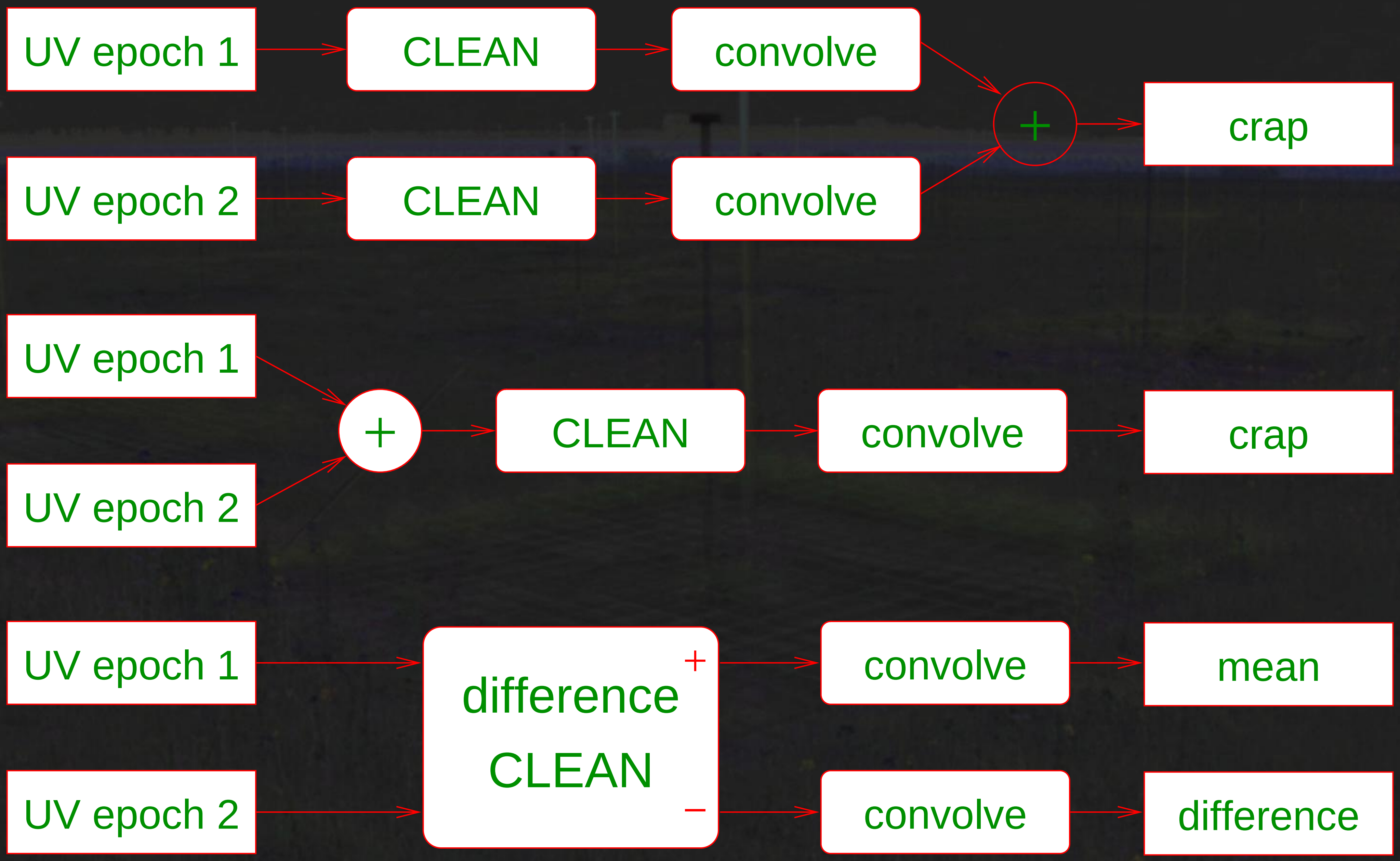



[ *Alard & Lupton (1998)* ]

# Radio difference imaging



# Related problem: combining different arrays



# Notation

- sky brightness distribution $\quad I(l,m) \quad \mathbf{I}$
- visibilities $\quad \tilde{I}(u,v) \quad \tilde{\mathbf{I}}$
- perfect measurement
  - ⋆ Fourier transform
    $$\tilde{I}_\mu = \sum_j A_{\mu j} I_j$$
    $$A_{\mu j} = \mathrm{e}^{2\pi \mathrm{i}(l,m)_j \cdot (u,v)_\mu}$$
  - ⋆ vectors
    $$\boxed{\tilde{\mathbf{I}} = \mathsf{A}\,\mathbf{I}}$$
- residuals
  $$\boxed{R^2 = (\mathsf{A}\mathbf{I} - \tilde{\mathbf{I}})^\dagger \mathsf{W} (\mathsf{A}\mathbf{I} - \tilde{\mathbf{I}})}$$
- natural weighting $\quad \mathsf{W} = \mathrm{diag}\left(\sigma_j^{-2}\right) \quad R^2 = \chi^2$

# Lazy interferometrists do it in image space

- expand $$R^2 = \tilde{\mathbf{I}}^\dagger \mathsf{W} \tilde{\mathbf{I}} + \mathbf{I}^\dagger \mathsf{A}^\dagger \mathsf{W} \mathsf{A} \mathbf{I} - 2 \mathbf{I}^\dagger \mathsf{A}^\dagger \mathsf{W} \tilde{\mathbf{I}}$$
- define (using $w = \mathrm{Tr}\,\mathsf{W}$)
  - ⋆ dirty beam $$\boxed{\mathsf{B} = \frac{\mathsf{A}^\dagger \mathsf{W} \mathsf{A}}{w}}$$
  - ⋆ dirty map $$\boxed{\mathbf{I}_{\mathsf{D}} = \frac{\mathsf{A}^\dagger \mathsf{W} \tilde{\mathbf{I}}}{w}}$$
- residuals derived in image space

$$\boxed{R^2 = \text{const} + \underbrace{w \left( \mathbf{I}^\dagger \mathsf{B} \mathbf{I} - 2 \mathbf{I}^\dagger \mathbf{I}_{\mathsf{D}} \right)}_{R'^2}}$$

minimum $\boxed{\mathsf{B}\mathbf{I} = \mathbf{I}_{\mathsf{D}}}$

# CLEAN as maximum likelihood fitting

- add components to the model $\mathbf{I}$
- minimize residuals in each step
- empty model plus component $I_j$ $\qquad R'^2 = I_j^2 - 2I_jI_{\mathrm{D}\,j}$
- optimal flux $\qquad \boxed{I_j = I_{\mathrm{D}\,j}}$
- residuals for optimal flux $\qquad \boxed{R'^2 = -I_{\mathrm{D}\,j}^{\,2}}$

$\rightsquigarrow$ optimal position: peak in dirty map

- subtract shifted beam from dirty map, start over

# Simultaneous CLEANing

- two epochs $\boxed{\tilde{\mathbf{I}}_1}$ and $\boxed{\tilde{\mathbf{I}}_2}$
- two models/maps $\boxed{\mathbf{I}_1}$ and $\boxed{\mathbf{I}_2}$
- combined residuals

$$\boxed{R^2 = \mathrm{const} + w_1 \left(\mathbf{I}_1{}^\dagger \mathsf{B}_1 \mathbf{I}_1 - 2\mathbf{I}_1{}^\dagger \mathbf{I}_{\mathrm{D}1}\right) + w_2 \left(\mathbf{I}_2{}^\dagger \mathsf{B}_2 \mathbf{I}_2 - 2\mathbf{I}_2{}^\dagger \mathbf{I}_{\mathrm{D}2}\right)}$$

- next component at same position in 1 and 2
  fluxes independent

$\rightsquigarrow$ optimal fluxes $\boxed{I_{\mathrm{D}1}}$ and $\boxed{I_{\mathrm{D}2}}$

- optimal position maximum of $\boxed{w_1 I_{\mathrm{D}1}^{\,2} + w_2 I_{\mathrm{D}2}^{\,2}}$

# Need for difference-CLEAN

- disadvantages of simultaneous CLEANing
  - ⋆ deconvolution errors still independent
  - ⋆ no control over mean and difference

- alternative approach
  two channels $\mathbf{I}_+$ and $\mathbf{I}_-$

$$\mathbf{I}_+ = \tfrac{1}{2}(\mathbf{I}_1 + \mathbf{I}_2)$$
$$\mathbf{I}_- = \tfrac{1}{2}(\mathbf{I}_1 - \mathbf{I}_2)$$

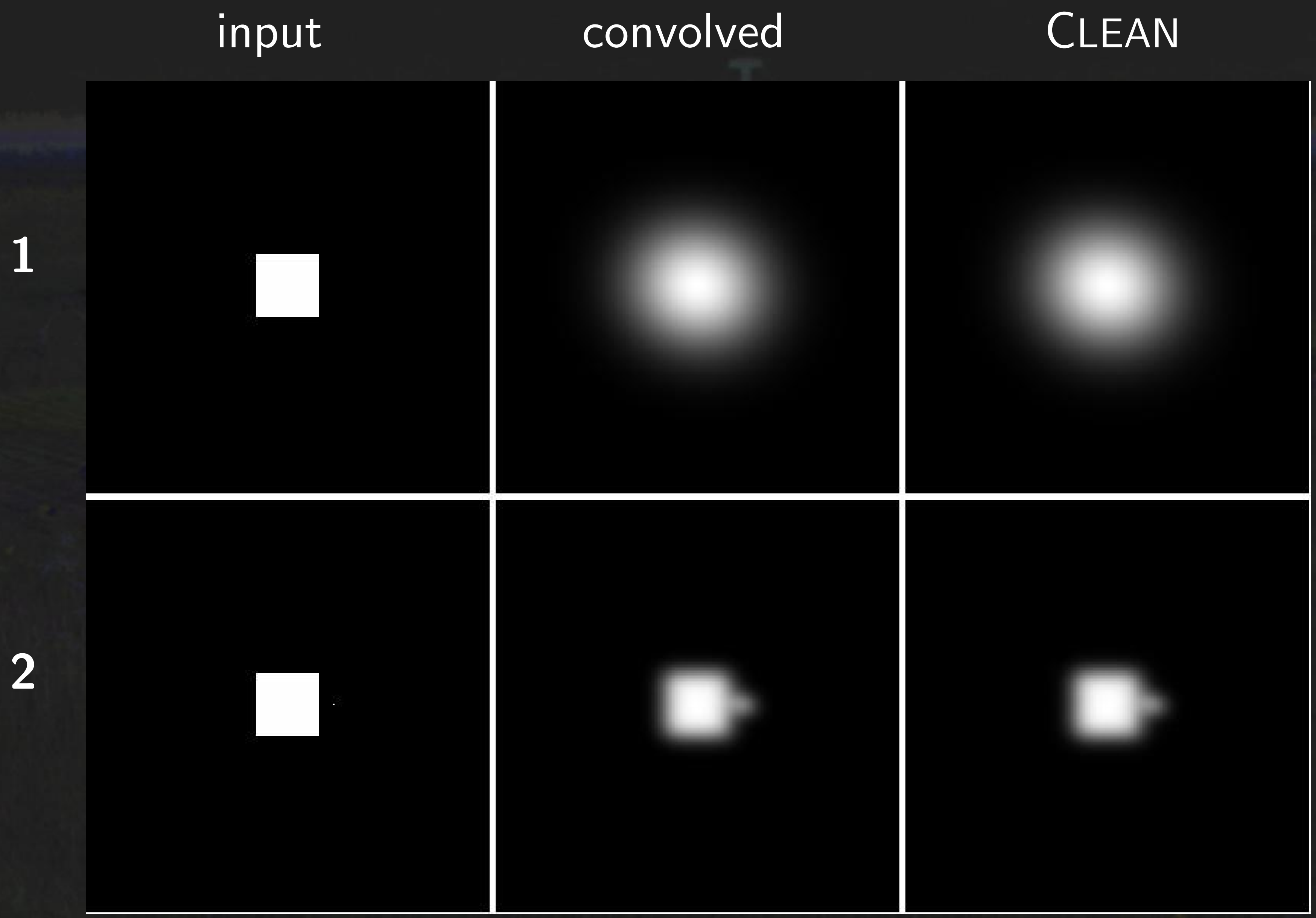
- dirty maps

$$\mathbf{I}_{\mathrm{D}+} = \frac{w_1\,\mathbf{I}_{\mathrm{D}1} + w_2\,\mathbf{I}_{\mathrm{D}2}}{w_1 + w_2}$$
$$\mathbf{I}_{\mathrm{D}-} = \frac{w_1\,\mathbf{I}_{\mathrm{D}1} - w_2\,\mathbf{I}_{\mathrm{D}2}}{w_1 + w_2}$$

# D-Clean procedure

- next component in either $\mathbf{I}_+$ or $\mathbf{I}_-$
- residuals $$R^2 = \text{const} - (w_1 + w_2) \times \begin{cases} {I_{\mathrm{D}+}}^2 & \text{for } + \\ {I_{\mathrm{D}-}}^2 & \text{for } - \end{cases}$$
- subtract according to $$\boxed{\begin{pmatrix} \mathbf{I}_{\mathrm{D}+} \\ \mathbf{I}_{\mathrm{D}-} \end{pmatrix} = \begin{pmatrix} \mathsf{B}_\parallel & \mathsf{B}_\times \\ \mathsf{B}_\times & \mathsf{B}_\parallel \end{pmatrix} \begin{pmatrix} \mathbf{I}_+ \\ \mathbf{I}_- \end{pmatrix}}$$
- $\mathbf{I}_+$ and $\mathbf{I}_-$ not independent
- dirty beams $$\boxed{\begin{aligned} \mathsf{B}_\parallel &= \frac{w_1 \mathsf{B}_1 + w_2 \mathsf{B}_2}{w_1 + w_2} \\ \mathsf{B}_\times &= \frac{w_1 \mathsf{B}_1 - w_2 \mathsf{B}_2}{w_1 + w_2} \end{aligned}}$$

# An experiment: VLA-like $uv$ coverage, scale 1:4

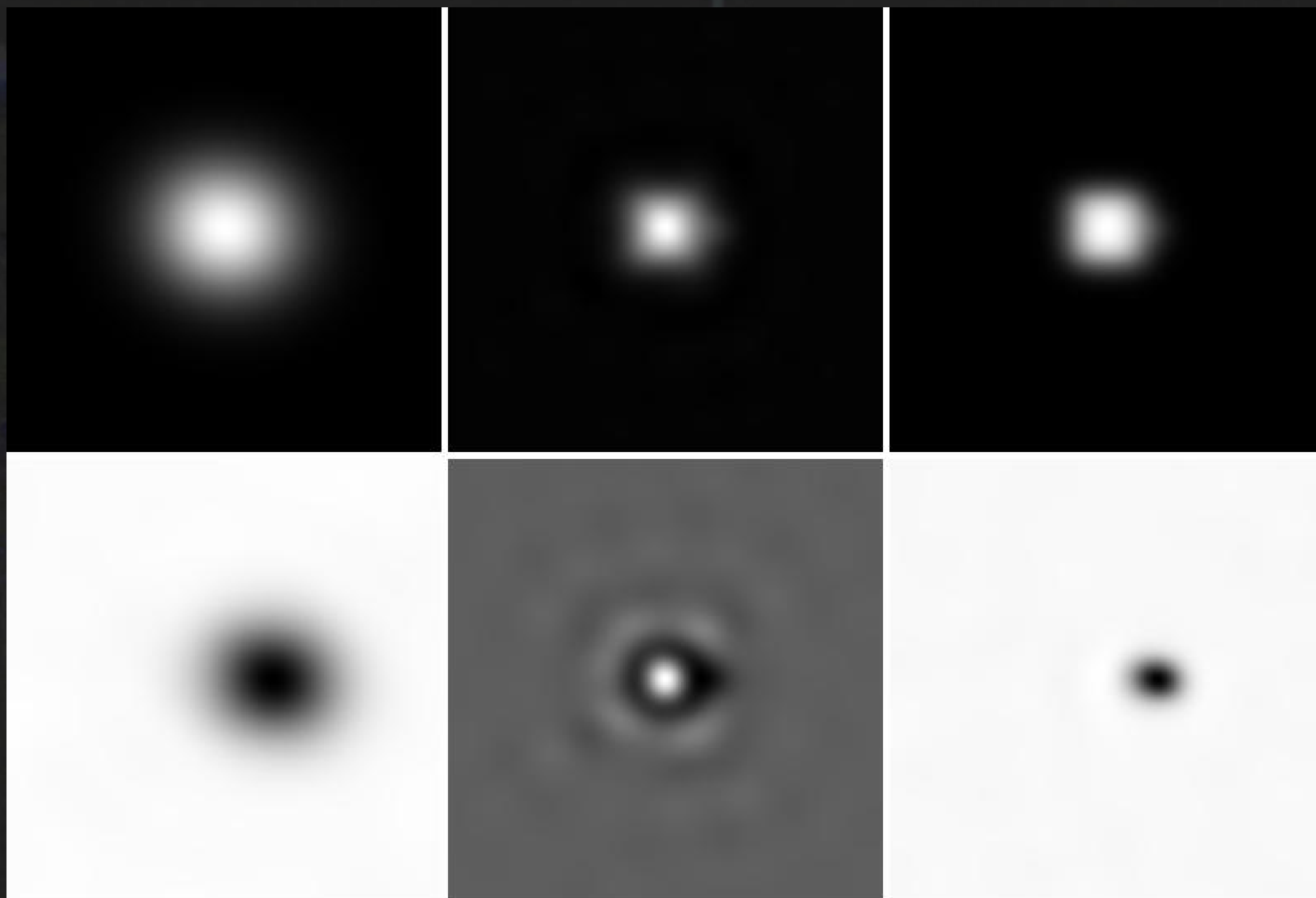


# Alternative methods

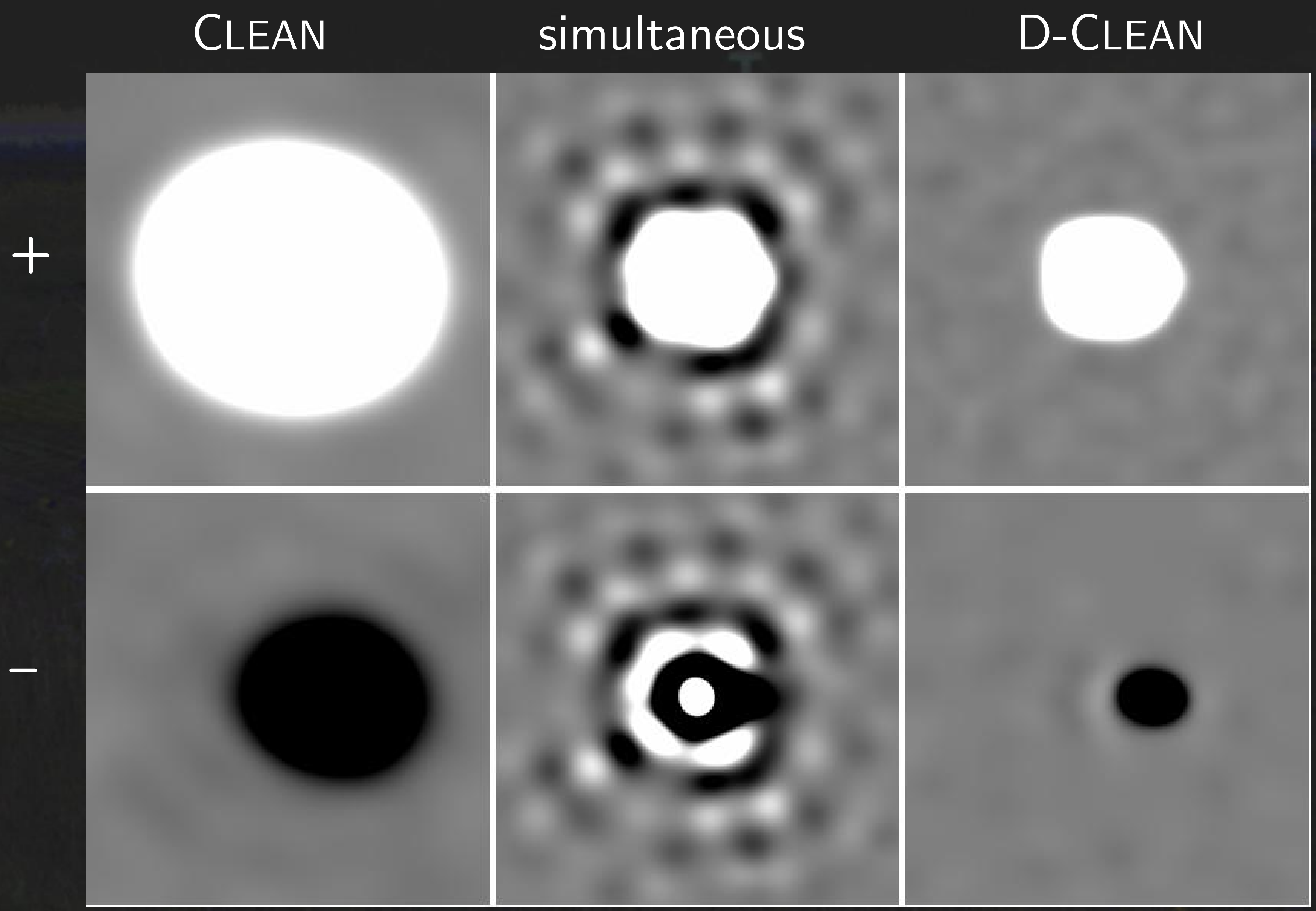
CLEAN simultaneous D-CLEAN

+

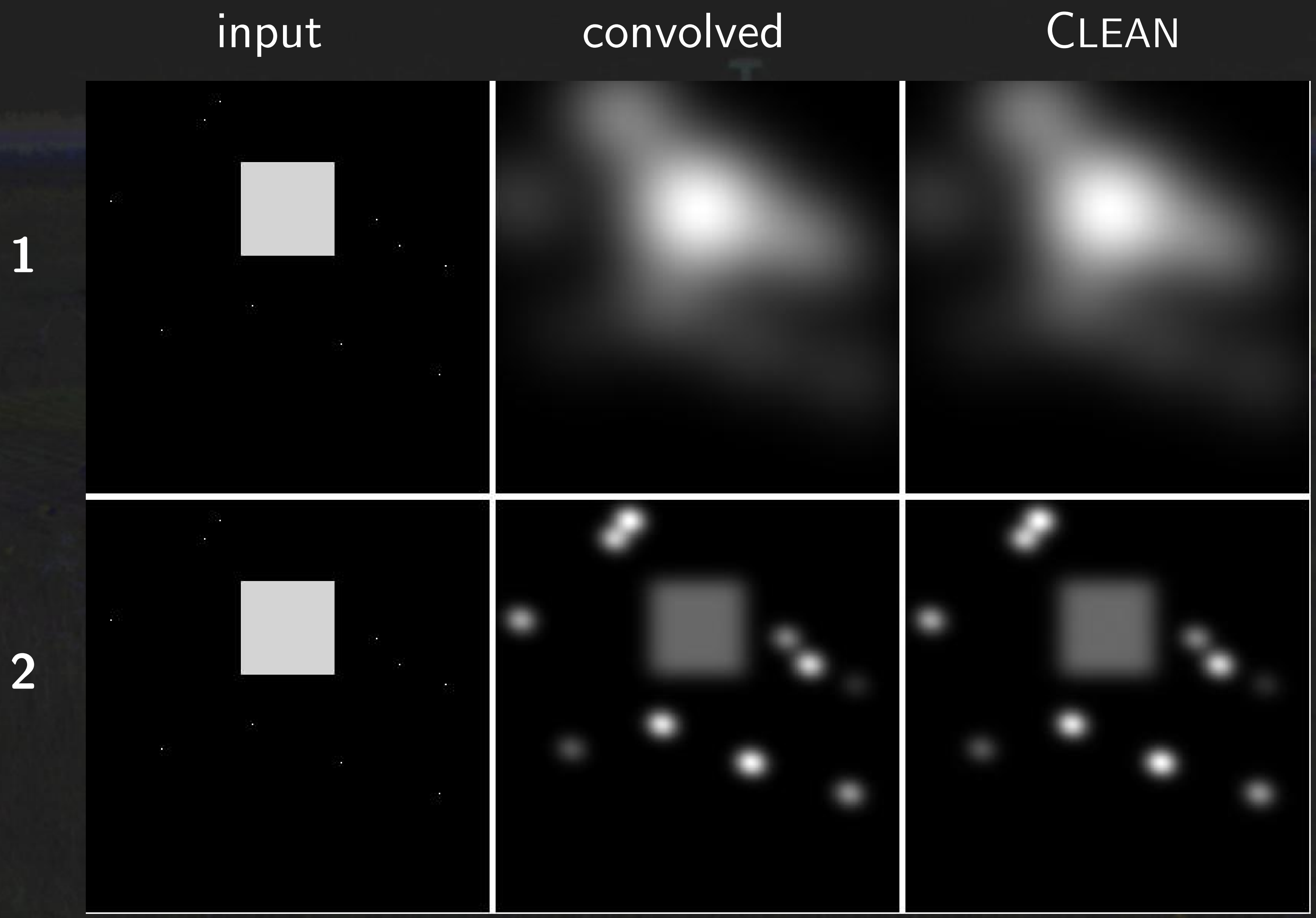
−



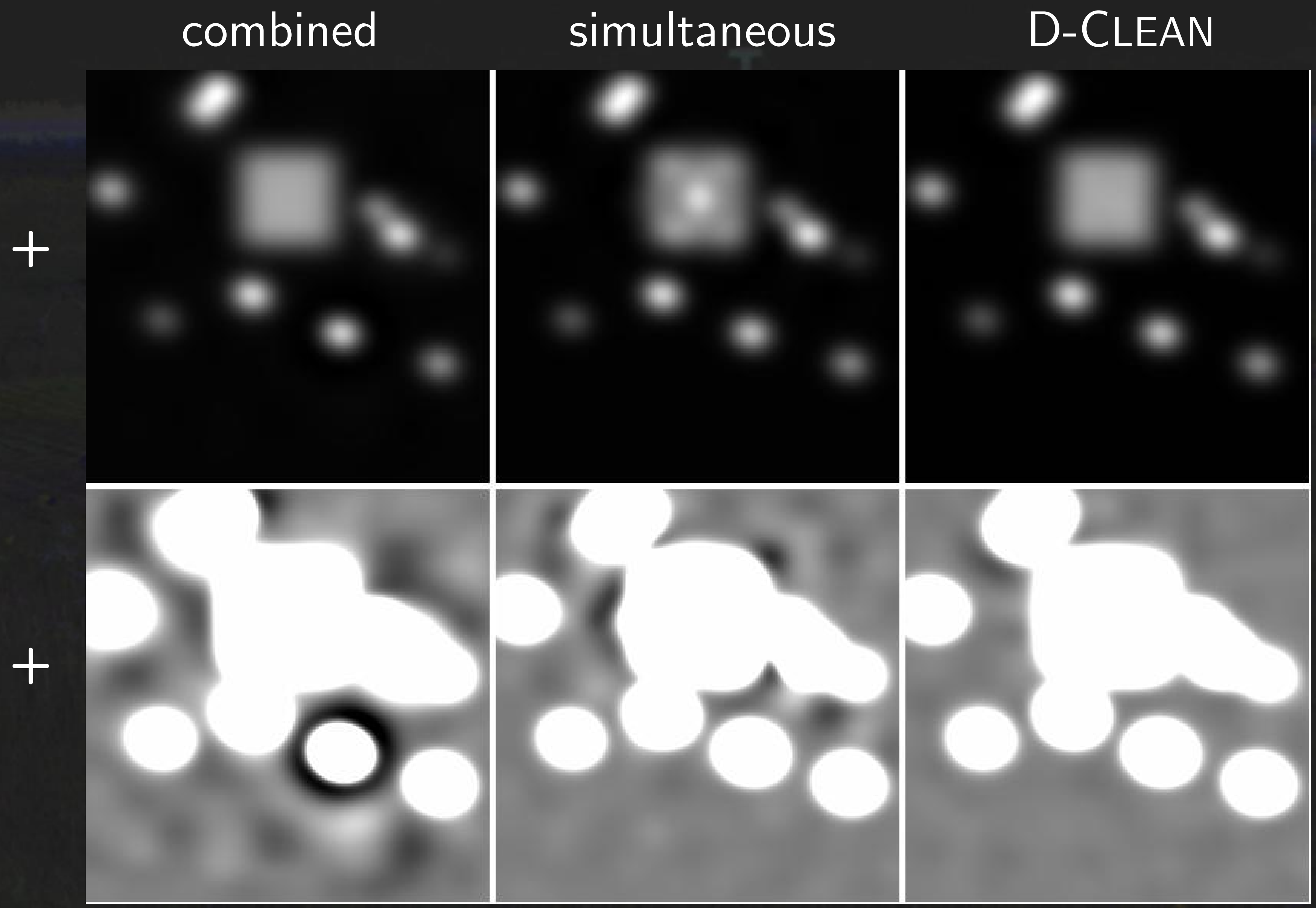
# Alternative methods: residual errors



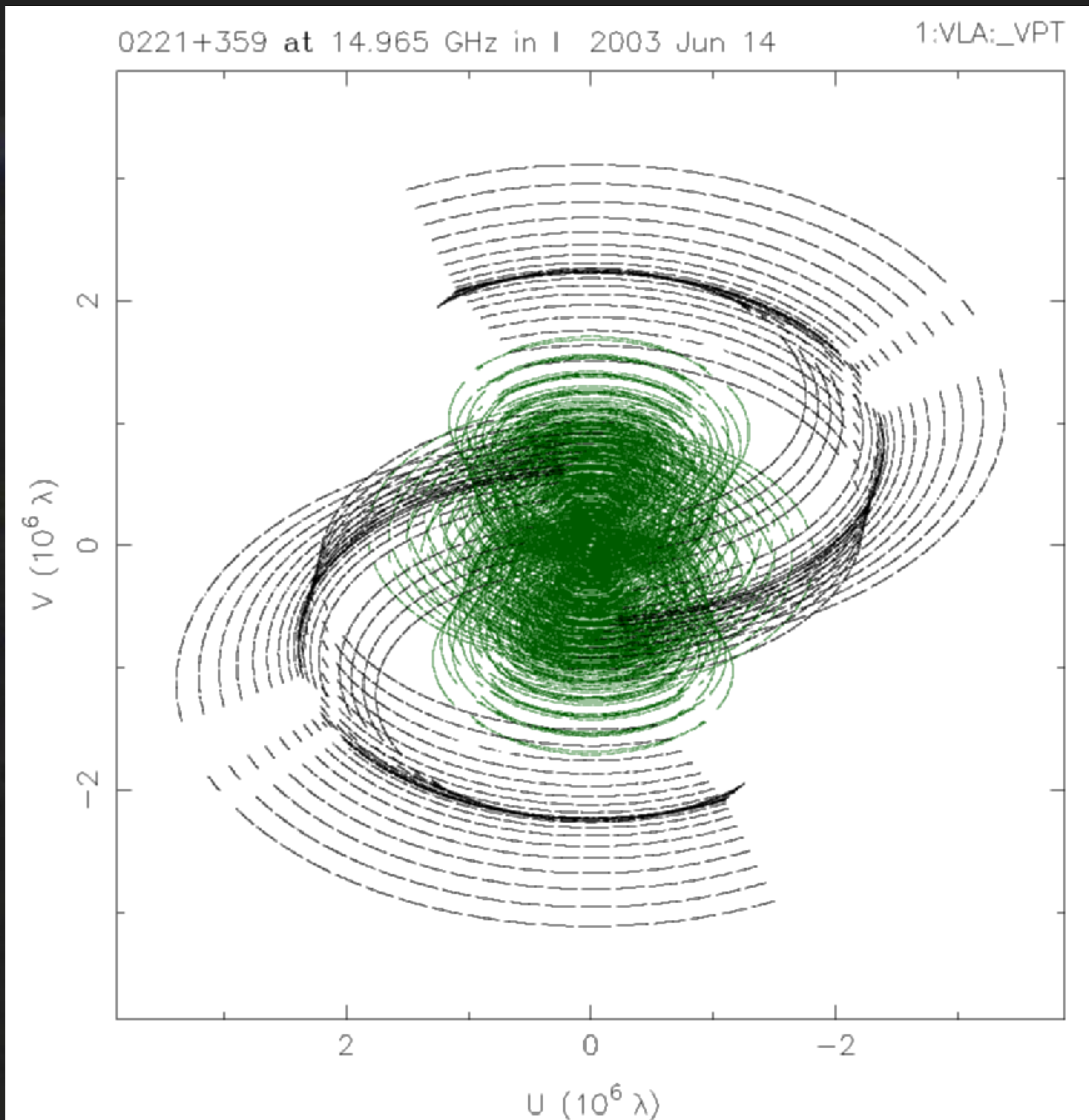
# Combining different arrays: input



# Combining different arrays: output



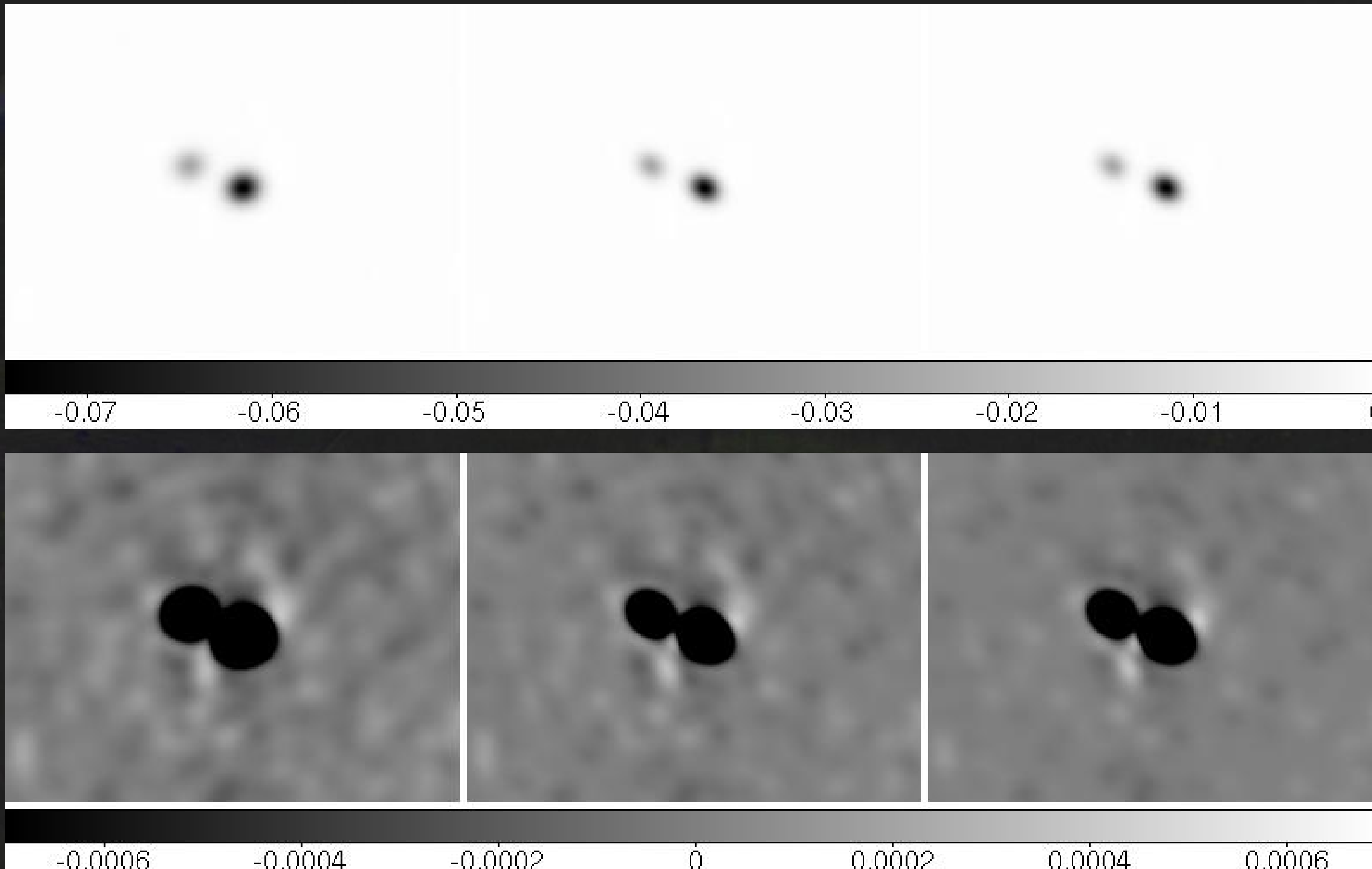
# Two-channel CLEANing of real data




- target lens B0218+357
  - ⋆ two bright images
  - ⋆ Einstein ring
- two VLA-A observations
  - ⋆ 1992
  - ⋆ 2003 with Pie Town
- use 1 IF

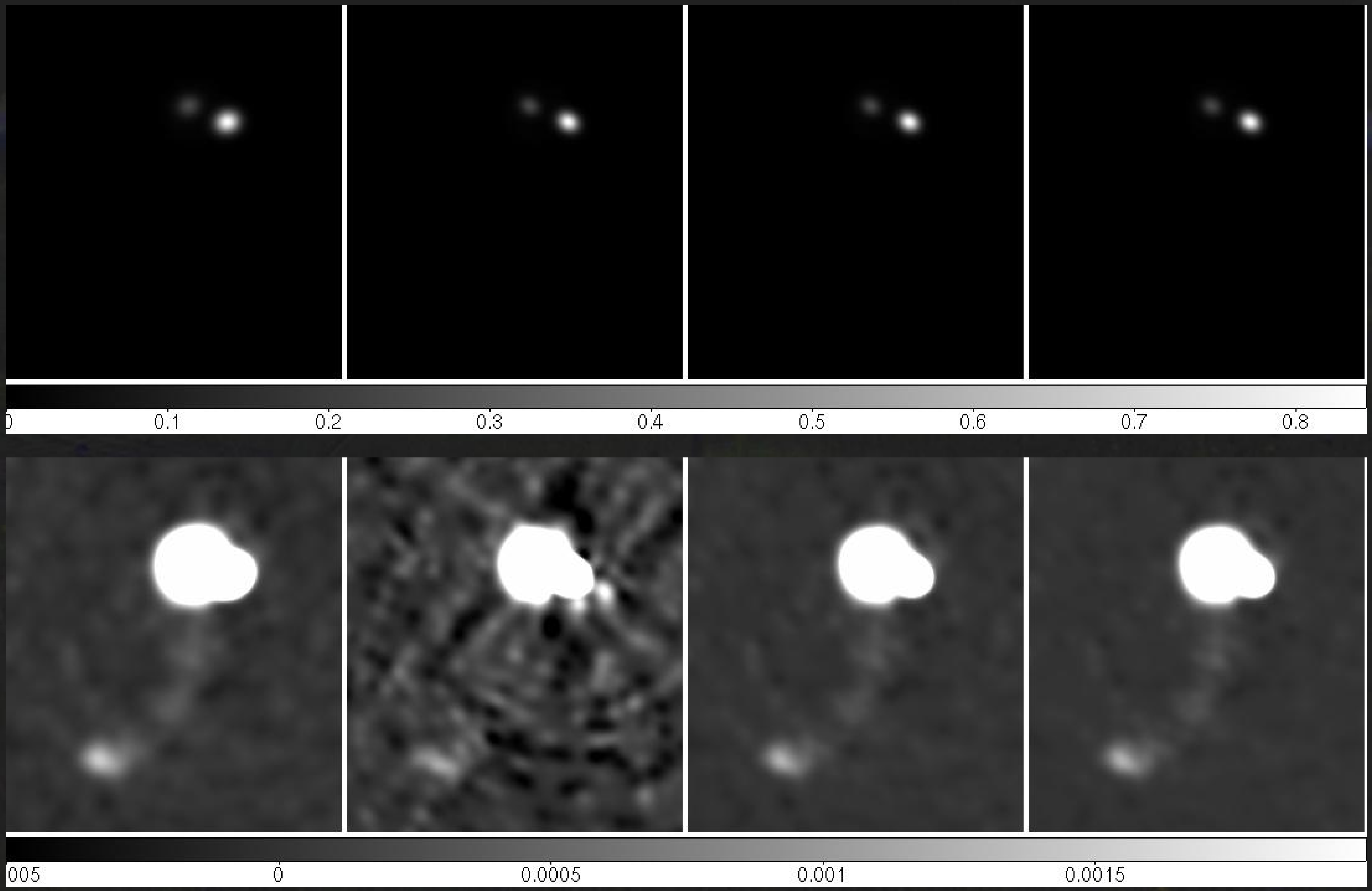
# Differencing images of B0218+357

CLEAN simultaneous D-CLEAN

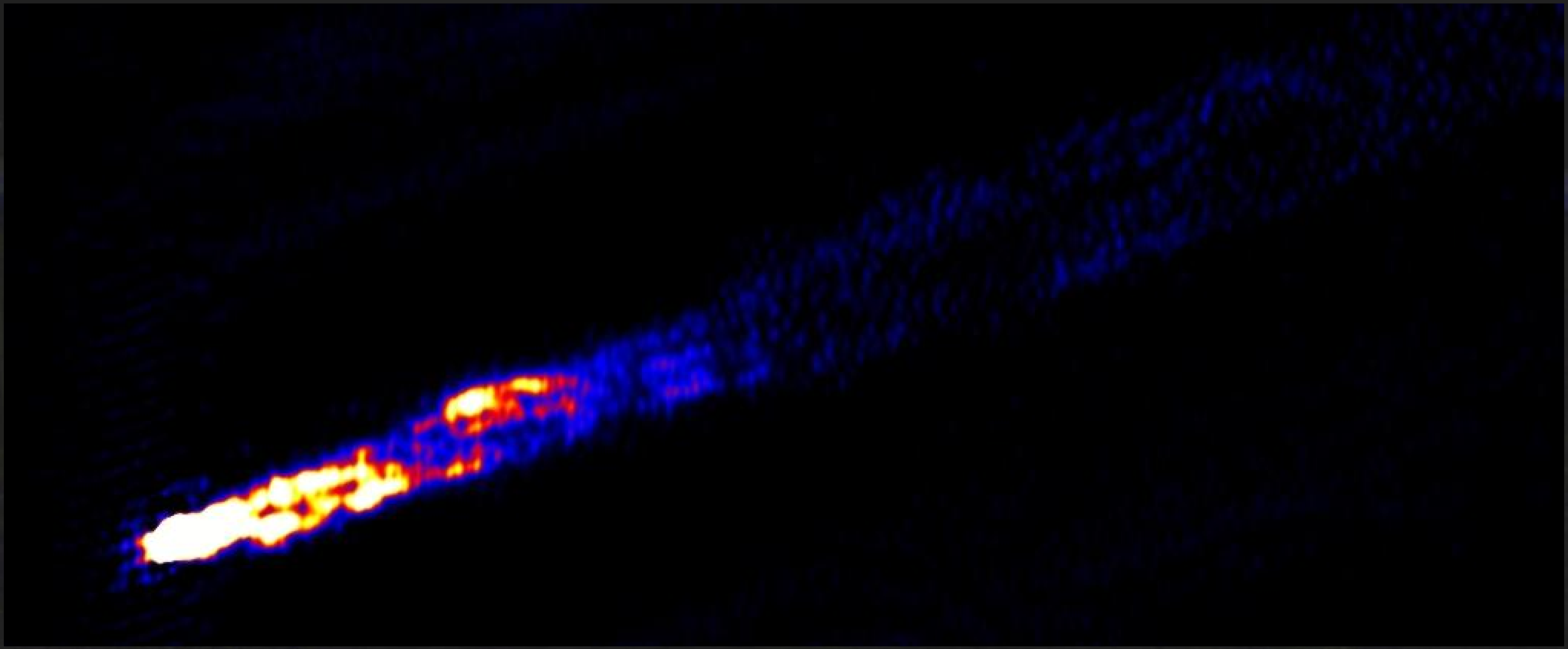


# Combined images of B0218+357

CLEAN combined simultaneous D-CLEAN



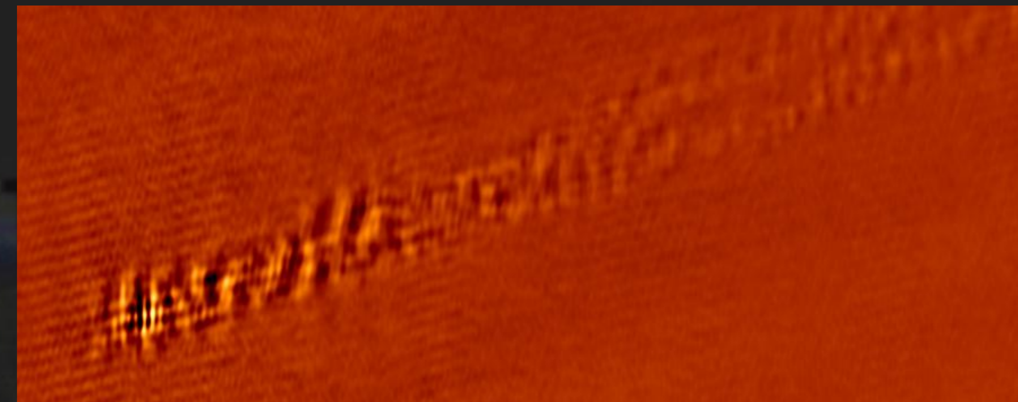
# Test with simulated VLBA data



- $uv$ coverage and model from real observations of Virgo A (thanks to Yuri Kovalev) [ details in *Kovalev et al. (2007)* ]
- following slides: simulation without/with noise, two epochs, slightly different $uv$ coverage

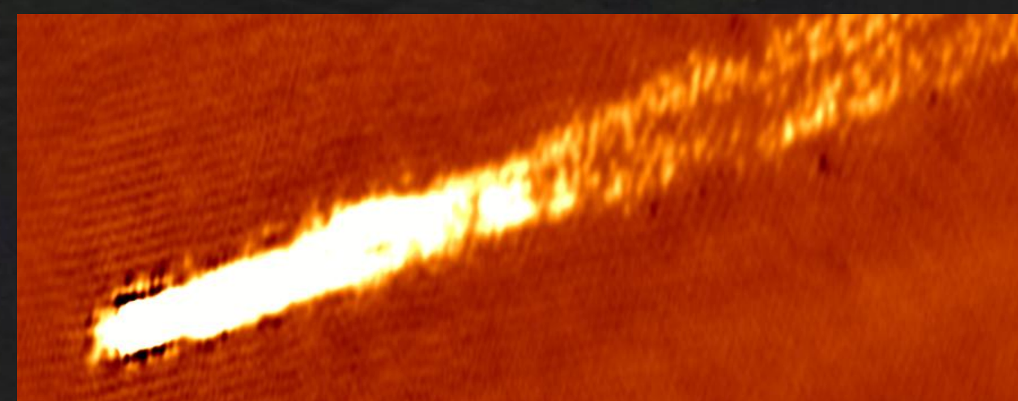
# Results for simulated data: no noise

difference of Clean maps



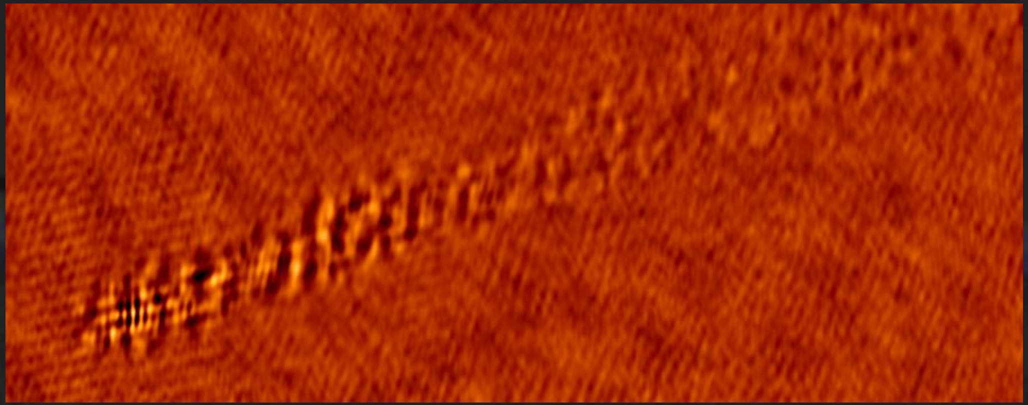
difference-Clean

mean Clean map

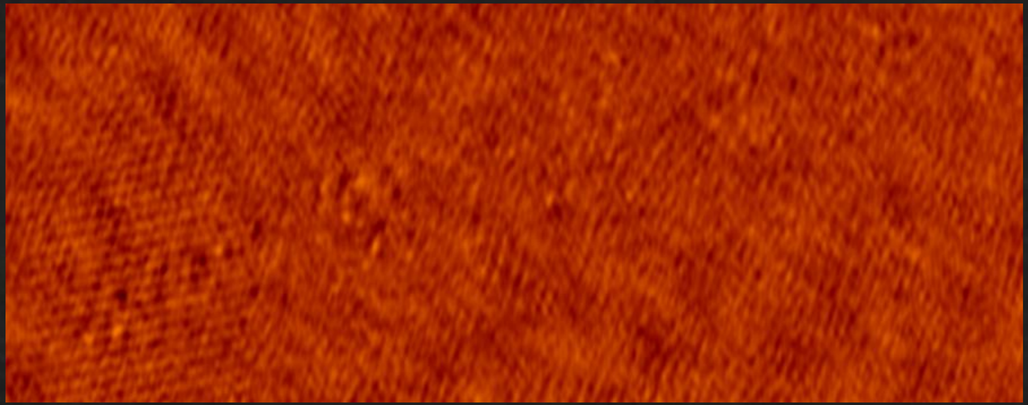


# Results for simulated data: with noise

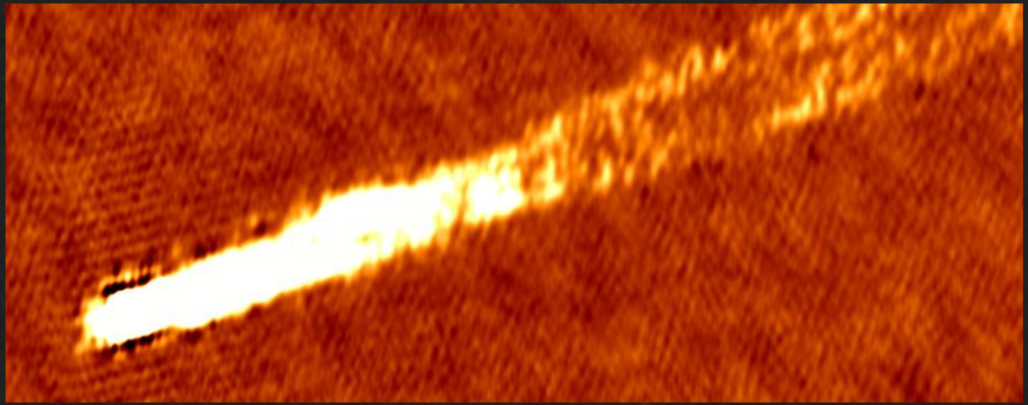
difference of Clean maps



difference-Clean



mean Clean map



# Alternative bases

- so far: two epochs
  - ⋆ constant part (sum)
  - ⋆ variable part (difference)
- many epochs
  - ⋆ constant part
  - ⋆ one additional part for each epoch
- continuous observation (or $\nu$ instead of $t$)
  - ⋆ constant part
  - ⋆ linear slope
  - ⋆ higher derivatives
- or general orthogonal polynomials, or . . .

# Constant + linear in frequency

- dirty maps

$$\mathrm{I_D}^{(n)} = \frac{\sum_\nu w(\nu)\,\nu^n\,\mathrm{I_D}(\nu)}{\sum_\nu w(\nu)}$$

- dirty beams

$$\mathsf{B}^{(n)} = \frac{\sum_\nu w(\nu)\,\nu^n\,\mathsf{B}(\nu)}{\sum_\nu w(\nu)}$$

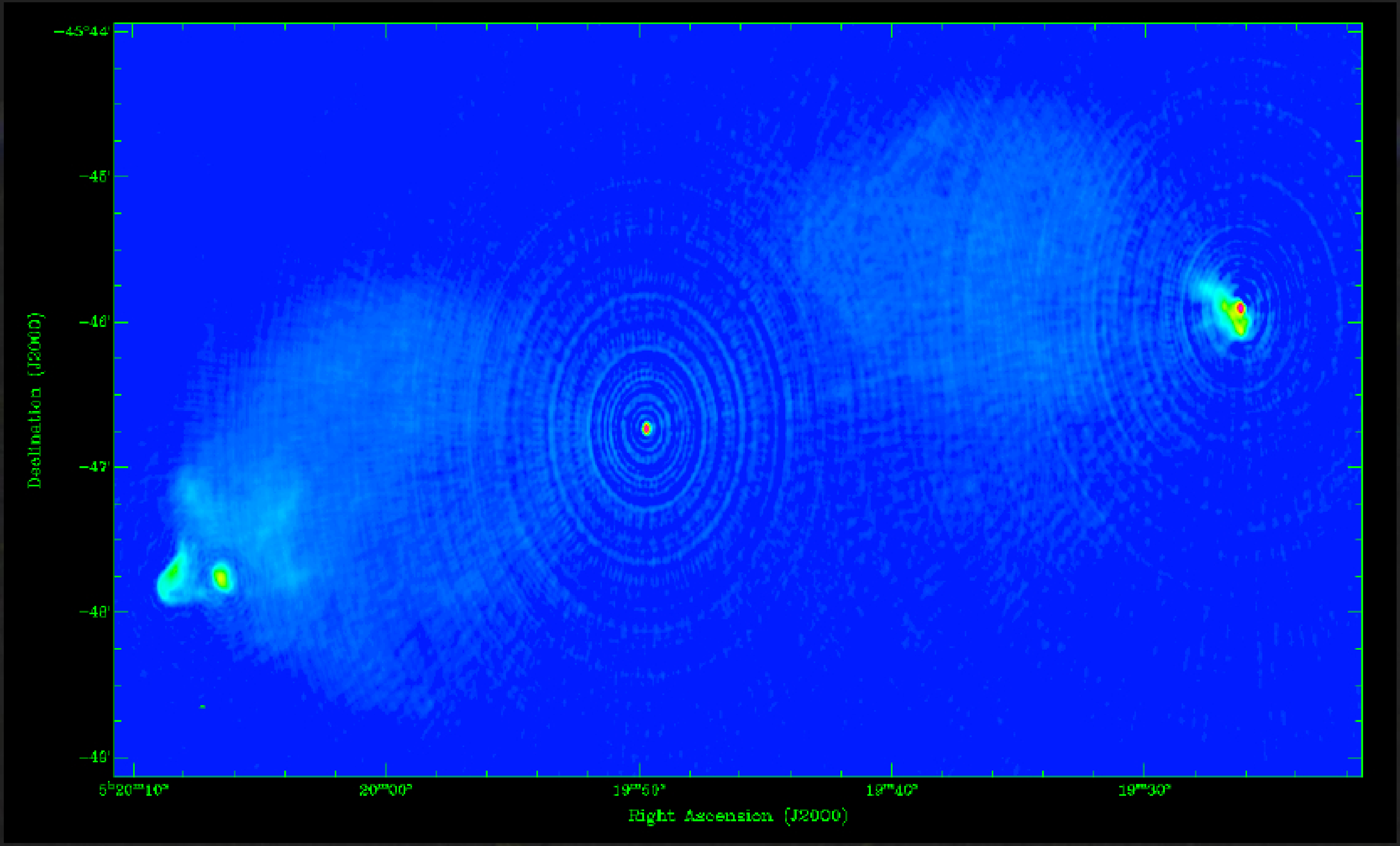
- convolution equation up to linear order

$$\begin{pmatrix}\mathrm{I_D}^{(0)}\\ \mathrm{I_D}^{(1)}\end{pmatrix} = \begin{pmatrix}\mathsf{B}^{(0)} & \mathsf{B}^{(1)}\\ \mathsf{B}^{(1)} & \mathsf{B}^{(2)}\end{pmatrix}\begin{pmatrix}\mathbf{I}^{(0)}\\ \mathbf{I}^{(1)}\end{pmatrix}$$

- previous MFS methods only use $\mathrm{I_D}^{(0)} = \mathsf{B}^{(0)}\mathbf{I}^{(0)} + \mathsf{B}^{(1)}\mathbf{I}^{(1)}$

  [ *Conway et al. (1990), Sault & Wieringa (1994)* ]

# ATCA observations of Pictor-A 4800 / 4928 MHz

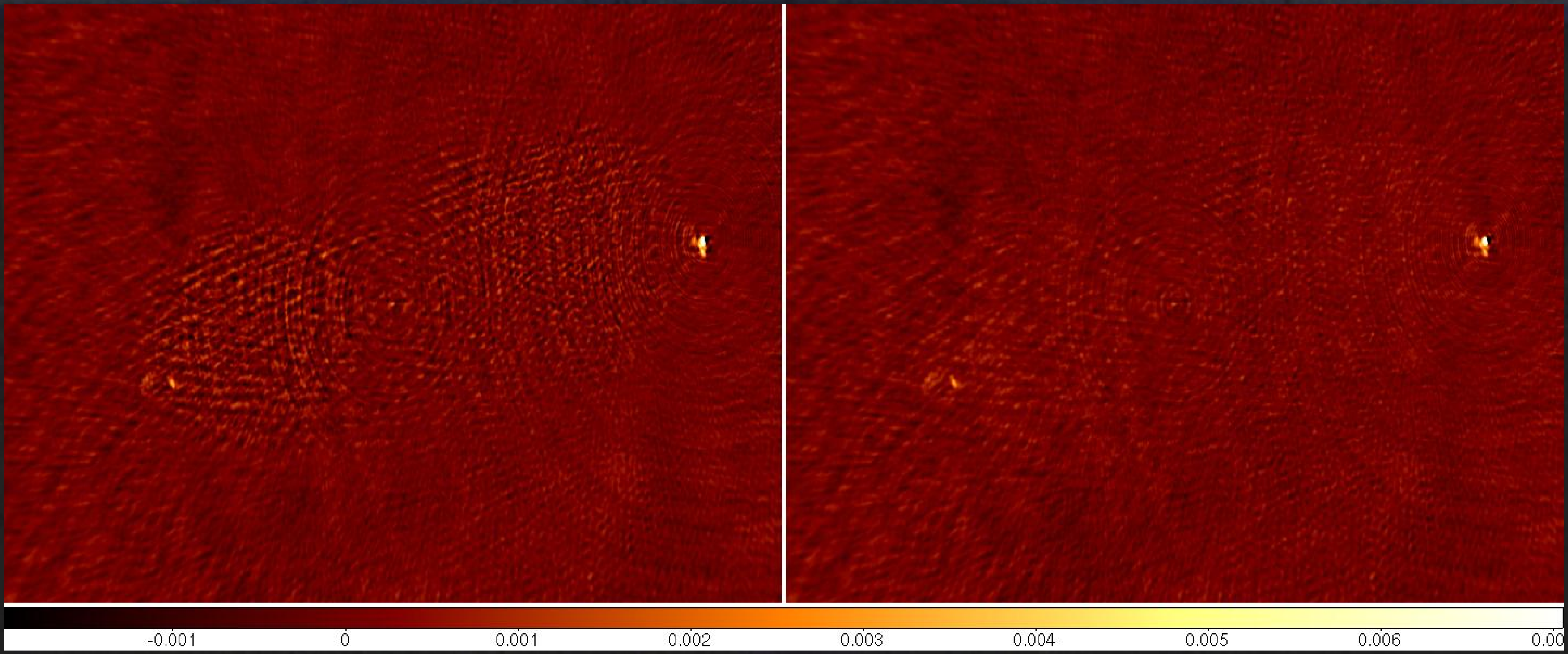



[ data from Emil Lenc ]

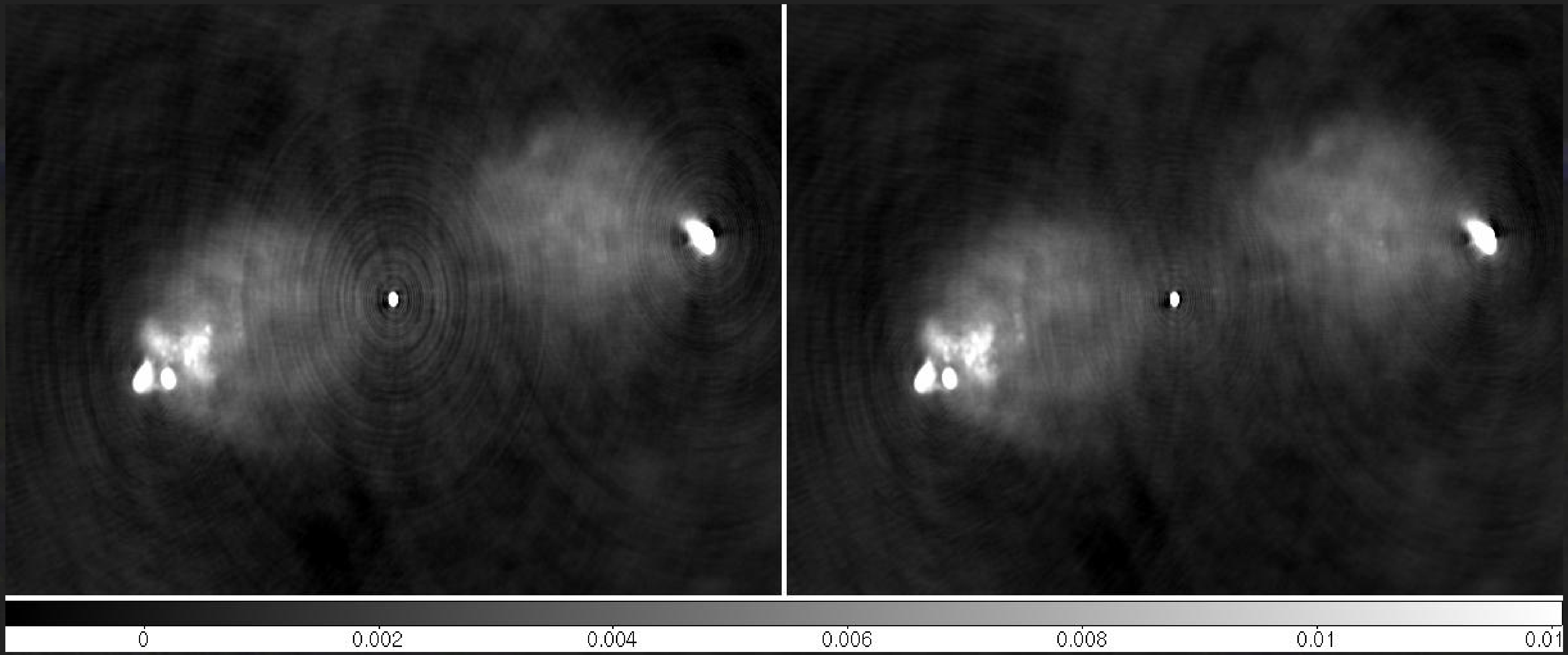
# Pictor-A: difference 4800 – 4928 MHz

difference of Clean maps

difference channel of D-Clean



# Pictor-A: correct for variability with AIPS (4800 MHz)




- five epochs with ATCA (different configurations)
- common model + variable parts

# Summary etc

- these tasks
  - ⋆ difference imaging, variability
  - ⋆ combine different arrays / epochs
  - ⋆ wide-band imaging with spectral indices
  - ⋆ multi-resolution / multi-scale Clean
- have in common that
  - ⋆ multi-channel output is needed
  - ⋆ channels do not always correspond to parts of the data
  - ⋆ want Clean regularisation for output channels
  - ⋆ need to deconvolve everything simultaneously
- developing multi-channel Clean
- first tests encouraging

work in progress

# Contents